KCSE PHYSICS PAPER 1: 2021 DISCUSSION

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NOTE: Questions have been re-arranged with questions from similar or related topics grouped together.

1. Figure 1 shows part of the thimble scale of a screw gauge with 50 divisions. On the diagram, draw the sleeve scale to show a reading of 3.87mm (1 mark)

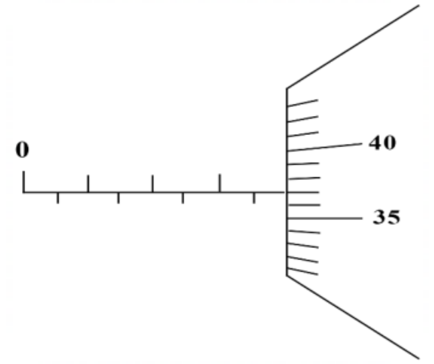


Figure 1

2. Figure 2 shows a siphon used to empty a tank.

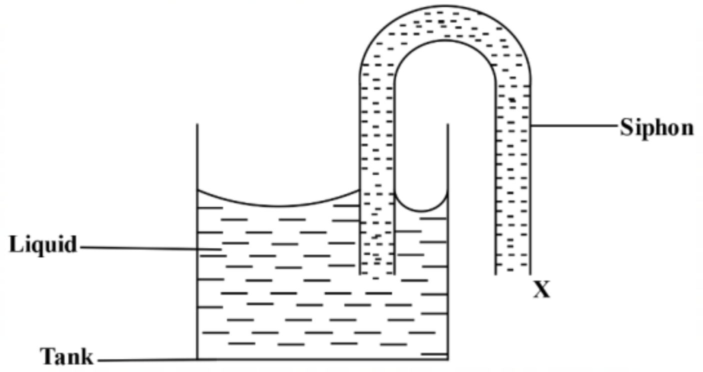


Figure 2

In order to start the siphon, state why:
(a) it must be full of liquid (1 mark)

Pressure exerted by solids

Pressure refers to the force a body exerts perpendicularly per unit surface area i.e.

$$Pressure\ (p) = \frac{Force\ (F)}{Area\ (A)} \qquad (i)$$

$$p = \frac{F}{A} \qquad (ii)$$

There are thus two factors that determine the pressure a solid exerts on a surface it is in contact with;

(i) Force (or weight); The more the force, the more the pressure and vice versa. If we consider two buddies, Biggi and Smolli, walking on the beach or on soft mud, and if Biggi with mass m_b, is heavier than Smolli with mass m_s, and assuming the area A of their feet in contact with the ground is equal, then, pressure p_b exerted by Biggi will be;

$p_b = \frac{m_b g}{A}$

Pressure p_s exerted by Smolli will be;

$P_h = \frac{m_s g}{A}$

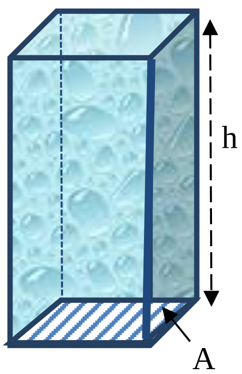
where g is the acceleration due to gravity.

Since $m_b > m_s$ it follows that $p_b > p_s$. Biggi will therefore sink deeper in the sand or in soft ground compared to Smolli.

(ii) Cross-sectional area; the smaller the cross-sectional area of contact, the greater the pressure and vice-versa. Someone in sharp pointed shoes sinks in soft mud or beach sand more than if he was wearing flat shoes. In nature, camels for example have broad hooves to prevent them from sinking in the soft sand, while elephants have flat 'feet' to prevent them from sinking in the ground owing to their massive weight.

Pressure exerted by liquids

Consider a closed rectangular tank of height h and cross-sectional area A. Suppose the tank is filled with a liquid, say water, of density ρ.



The pressure p the column of water exerts on the base of the tank is given by:

$$p = \frac{F}{A}$$

Now, the force F is equal to the weight W of the water column. If m be the mass of the water in the tank and g the acceleration due to gravity, then:

$$F = W = mg \quad \text{(iii)}$$

Hence;

$$p = \frac{mg}{A} \quad \text{(iv)}$$

From definition of density and considering V to be the volume of water in the tank, it follows that;

$$m = \rho V \quad \text{(v)}$$

But;

$$V = Ah \quad \text{(vi)}$$

Hence;

$$m = \rho Ah \quad \text{(vii)}$$

Using equation (vii) in equation (iv) leads to;

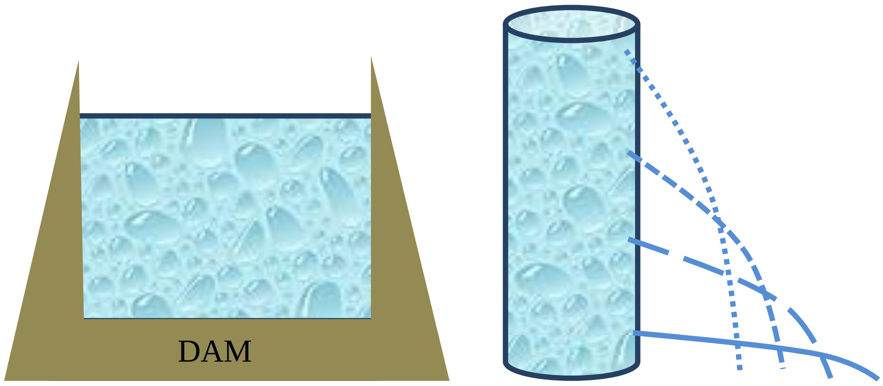
$$p = \frac{\rho Ahg}{A}$$

$$p = \rho gh \quad \text{(viii)}$$

Equation(viii) is used to evaluate pressure exerted by liquids and gases (fluids). It shows that the pressure exerted is independent of the surface area. It however depends on:

(i) Density; The pressure exerted by a fluid is directly proportional to its density. For example, a column of water exerts more pressure on the base of a container compared to a column of paraffin of equal dimensions. A column of mercury on the other hand exerts more pressure than a column of water of equal dimensions. Mercury has a density of 13,546 kg/m^3, water 1000 kg/m^3 and oil (paraffin) 900 kg/m^3.

(ii) Height of the fluid column (depth); Pressure exerted by a fluid column increases with depth below the surface. For example, dams are constructed with thicker walls near the base to enable them withstand the relatively higher pressure. Additionally, if for example a long tube with vertical holes on the side is filled with water, the length of waterjet from the bottommost hole will be the longest due to the relatively high pressure, with the length of the waterjets gradually reducing as the holes get closer to the surface.



Atmospheric pressure

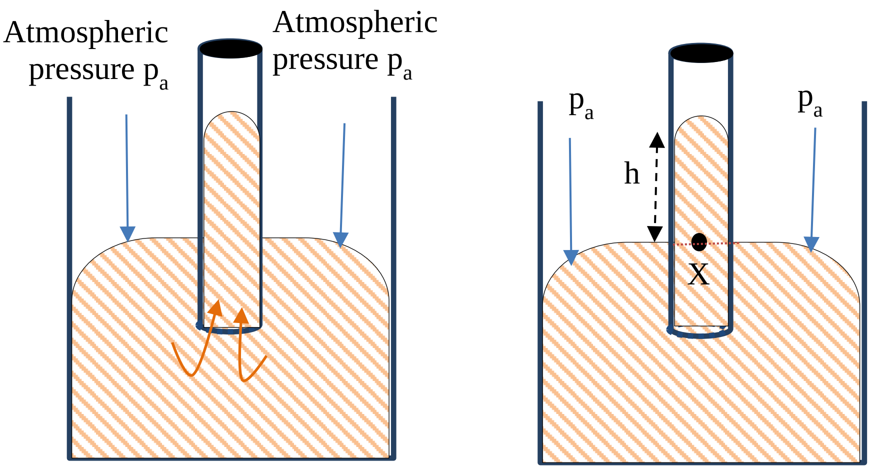
As weird as it may sound, air has mass and therefore weight (force) as a result of gravitational attraction. The vertical force exerted on a surface by an air column as a result of gravitational attraction is referred to as atmospheric pressure or air pressure.

The pressure exerted by an air column of height h is given as;

$p = \rho g h$

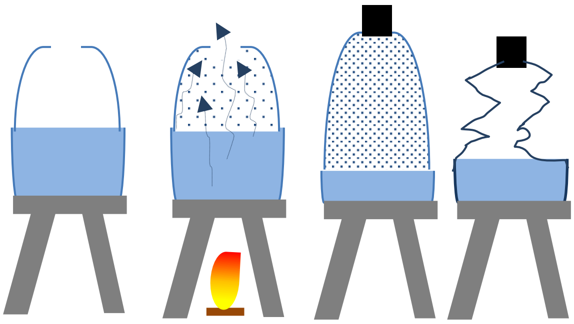
Where ρ is the density of the air column. The density of air on the earth's surface is not uniform. At the sea level for instance, air molecules are more concentrated but as the elevation (altitude) increases, the concentration reduces. This means that density of air is higher at lower altitudes (for example Mombasa) compared to higher altitudes (for example Nairobi). Since pressure is directly proportional to density, it follows that atmospheric pressure reduces with altitude. Atmospheric pressure is maximum at the sea level with a value of 101,325 Pa.

Atmospheric pressure is commonly measured using a barometer. A barometer is simply a column of mercury in a glass tube inverted in an open vessel containing more mercury. The length of the mercury column is used to determine the magnitude of the atmospheric pressure. To make a simple mercury barometer, an evacuated tube (tube with air pumped out) is placed upside down in an open glass bowl containing mercury. The atmosphere exerts pressure on the surface of the mercury in the bowl but not in the tube given that it is sealed at the top. Considering that the tube is nearly at zero pressure, the higher atmospheric pressure pushes the mercury into the tube.



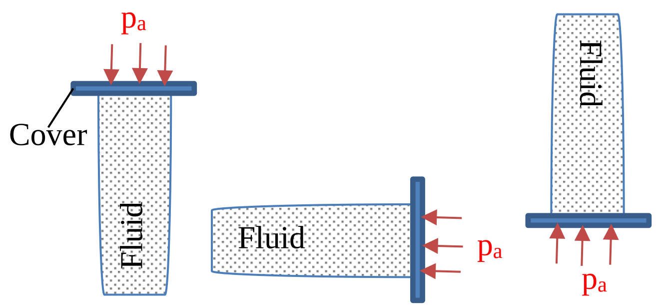
This ceases when the pressure the mercury column exerts at point X, which is at the same level as the mercury in the bowl, is equal to the atmospheric pressure. At sea level, the length of mercury the atmosphere can hold at normal temperature is 760 mm. Atmospheric pressure is therefore often given as 760 mmHg (760 mm of mercury). It is important to note that the atmosphere can hold longer columns of less dense liquids. For example, the atmosphere can hold a water column with a vertical height of up to 10.3 m.

An experiment normally used to demonstrate the existence of atmospheric pressure is the crashed can experiment. In the experiment, a metal can is partially filled with water and heated while open. Once the water starts boiling, the can is sealed, immediately removed from the heat, then left to cool down (or cold water poured on it to hasten the cooling process). It is observed that the metal can crashes as it cools down.



The reason for this is that as the water heats up, the steam generated drives out the air in the metal can and takes its place and all through, the atmospheric pressure is equal to the pressure inside the open can. When the can is sealed and allowed to cool down, the vapour condenses (becomes water) thereby leaving a partial vacuum in the sealed can. At this point, the atmospheric pressure is greater than the pressure inside the can hence exerts force on the can crashing it in the process. It is important to note that atmospheric pressure (p_a) acts equally in all directions.

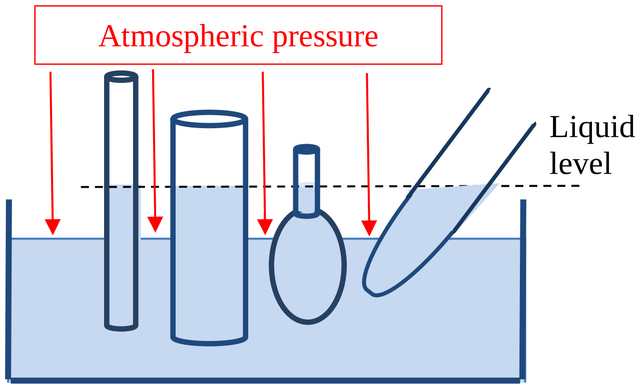
To further illustrate the presence of atmospheric pressure, a glass tumbler is filled with a fluid, say water, and covered with a flat cardboard. While supporting the cardboard, the can is turned upside down after which the support is withdrawn.



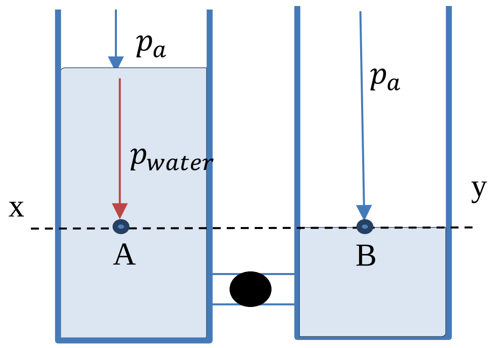
The water column, say of density ρ and height h, remains in the tumbler so long as the pressure it exerts on the cardboard cover is less than the atmospheric pressure i.e., $\rho g h < p_a$.

NOTE 1: The higher the pressure deference, the greater the impact. A can with less water would crash more as more air would have been pushed out by water vapour.

NOTE 2: Pressure in a liquid is the same at all points at that level. For example, if tubes of different shapes, sizes, and orientation are dipped in a liquid, the rise of the liquid in all the tubes on account of atmospheric pressure will rest at the same horizontal level.



NOTE 3: Suppose two open containers are connected via a tube with a tap. Suppose too that with the tap closed, one of the containers has more water than the other.

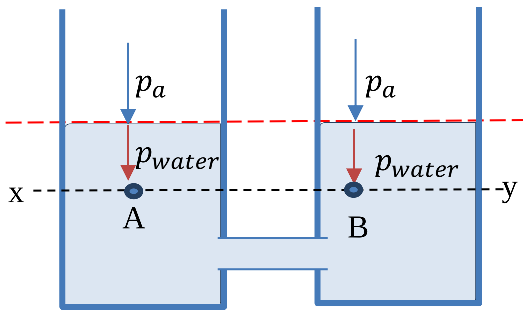


The pressure at A (p_A) equals the atmospheric pressure (p_a) plus the pressure due to the water column (p_{water}) while the pressure at B (p_B) equals the pressure exerted by the atmosphere only, i.e.,

$p_A = p_a + p_{water}$ (i)

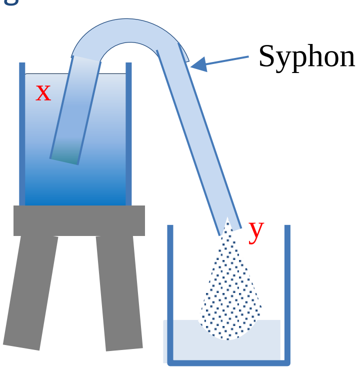
$p_B = p_a$ (ii)

When the tap in the connecting pipe is opened, water flows from container A to container B on account of the pressure difference. The flow ceases when the pressure at points A and B are equal. This happens irrespective of the shape of the containers. This means that the pressure a liquid (or fluid) exerts at all points on the same horizontal level (xy) is equal.



NOTE 4: Syphon

A syphon can be used to drain water from a tank. A syphon is a tube that allows a liquid to move up the tube from a source, over the bend of the tube and down the longer limb.

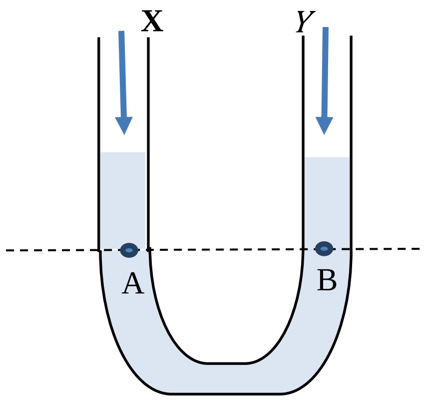


The water level (x) must however be at a higher level than the outlet (y) and the syphon be filled with water at the start of the siphoning. A combination of factors that include atmospheric pressure, gravity and cohesive force between the water molecules is said to be responsible for the movement of water (or any other liquid) through a syphon.

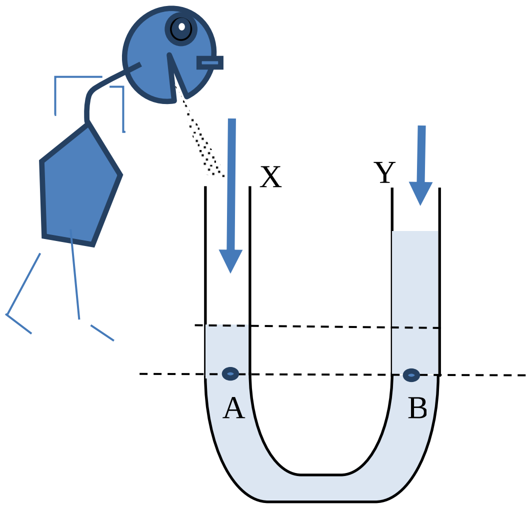
NOTE 5: U-tube manometer

That pressure in a liquid is equal at all points in a given level is behind the working principle of a U-tube manometer, a device used for measuring pressure or comparing densities of two liquids. A U-tube manometer is simply a U-tube containing two immiscible liquids (liquids that do not mix) with the height of the liquid columns being used to determine the relative density of, or pressure exerted by, the liquid of interest.

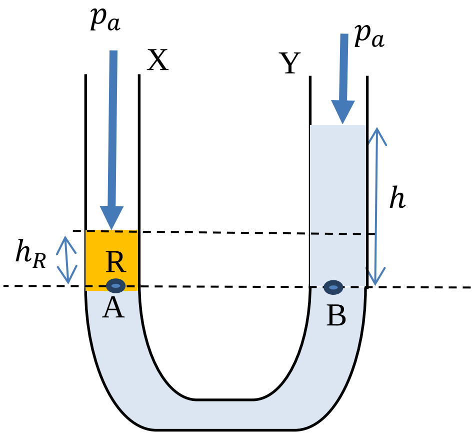
Suppose a U-tube is partially filled with some liquid, say water. The water attains the same level in both tubes, X and Y.



This means that points A and B are at the same pressure (equal to atmospheric pressure and pressure of the water column above the points). If air is blown into tube X, pressure at A exceeds that at B. Consequently, water will be pushed down tube X and up tube Y until the pressure at A balances that at B.



Suppose now instead of blowing into tube X, an immiscible liquid R of density ρ_R is gradually poured into the tube until it touches point A.



If the height of liquid R column be h_R, the pressure at A (p_A) is equal to the sum of the atmospheric pressure and the pressure exerted by the liquid R column, i.e.

$p_A = p_a + g\rho_R h_R$ (i)

Pressure at B (p_B) equals the sum of the atmospheric pressure and the pressure exerted by the water column above it. If ρ be the density of water and h the height of the water column, then;

$p_B = p_a + g\rho h$ (ii)

Considering that pressure at A is equal to the pressure at B, it follows that;

$p_a + g\rho_R h_R = p_a + g\rho h$ (iii)

$\rho_R h_R = \rho h$ (iv)

Since $h_R < h$, it follows that less of liquid R is required to give the same pressure as water. This means that the density of liquid R is greater than that of water.

NOTE 6: Applications of atmospheric pressure

- Syringe when used to draw a liquid: When the syringe piston is pushed down (before use) air in the syringe is pushed out. When the piston is pulled up (during use) pressure inside the syringe reduces and the higher atmospheric pressure pushes the liquid up into the syringe.
- Straw: When a person sucks through a straw, the pressure in the straw reduces and the higher atmospheric pressure pushes the liquid up. A straw with a hole on the side cannot be used to draw up a liquid as air sucked out is balanced by air getting in through the hole. The pressure in the straw is therefore always equal to the atmospheric pressure.
- Vacuum cleaner: When a vacuum cleaner is switched on, air is sucked out from inside the cleaner thereby reducing the pressure inside. The higher atmospheric pressure then forces air and dust particles into the cleaner.

NOTE 3: Pascals principle

Pascal's principle (Pascal's law) states that for a liquid at rest and enclosed in a container, pressure applied at one point is equally transmitted throughout the liquid. Pascal law is applied for instant in a hydraulic press. The press is made up of a u-tube of different cross-sectional areas A_1 (effort piston) and A_2 (load piston) with $A_1 < A_2$. When a force F_1 is applied in the smaller tube, the pressure p generated is given by;

$$p = \frac{F_1}{A_1} \qquad \text{(i)}$$

This pressure is distributed evenly throughout the liquid and consequently, a force F_2 is generated at the load piston such that;

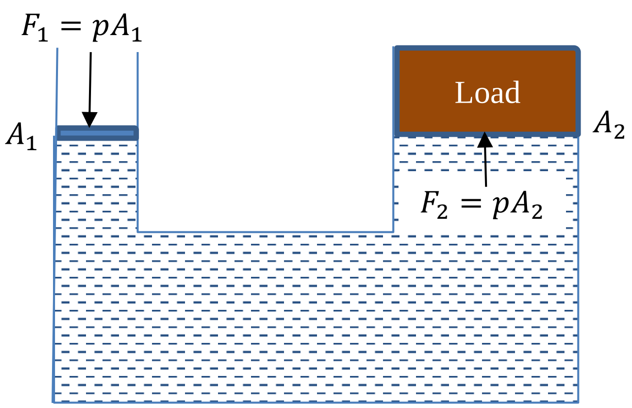
$$p = \frac{F_2}{A_2} \qquad \text{(ii)}$$

Equation (ii) implies that if A_2 is large, F_2 has to be equally large so as to keep the pressure constant.

Equations (i) and (ii) can be combined into a single equation as;

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \text{(iii)}$$

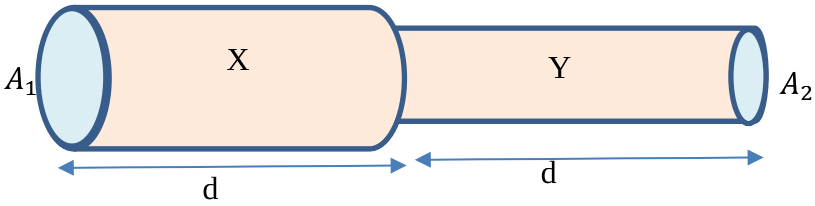
A small force at the effort piston generates a significantly large force at the load piston hence the load is easily compressed.



The same principle is also applied in hydraulic lift where a small amount of force (effort) generates a larger force that is used to lift a large load.

Continuity equation and Bernoulli's Principle:

Continuity equation is concerned with transport and conservation of mass. Imagine water flowing steadily in a pipe of different cross-section areas as shown. Let A_1 and A_2 be the cross-sectional areas of the wider (X) and narrower (Y) pipes, respectively. Suppose the two pipes are of equal length d.



If in time t_1 the wider pipe (X) is filled with water, then all that water must pass through the narrower pipe (Y) before X can fill-up with 'new' water again. This means that the volume flow rate (volume of water divided by time) of water in pipe Y must be equal to the volume flow rate of water in pipe X.

$Volume\ flow\ rate\ pipe\ X = Volume\ flow\ rate\ pipe\ Y$ (i)

If all the water that passed through pipe X in time t_1 passes through pipe Y in time t_2, then;

$Volume\ flow\ rate\ in\ pipe\ X = \frac{volume\ in\ pipe\ X}{t_1} = \frac{A_1 d}{t_1} = A_1 \frac{d}{t_1}$ (ii)

But $\frac{d}{t_1} = v_1\ (speed\ of\ water\ in\ X)$ hence;

$Volume\ flow\ rate\ pipe\ X = A_1 v_1$ (iii)

$Volume\ flow\ rate\ pipe\ Y = \frac{volume\ in\ pipe\ Y}{t_2} = \frac{A_2 d}{t_2} = A_2 \frac{d}{t_2}$ (iv)

But $\frac{d}{t_2} = v_2$ (speed of water in Y) hence;

$Volume\ flow\ rate\ pipe\ Y = A_2 v_2$ (v)

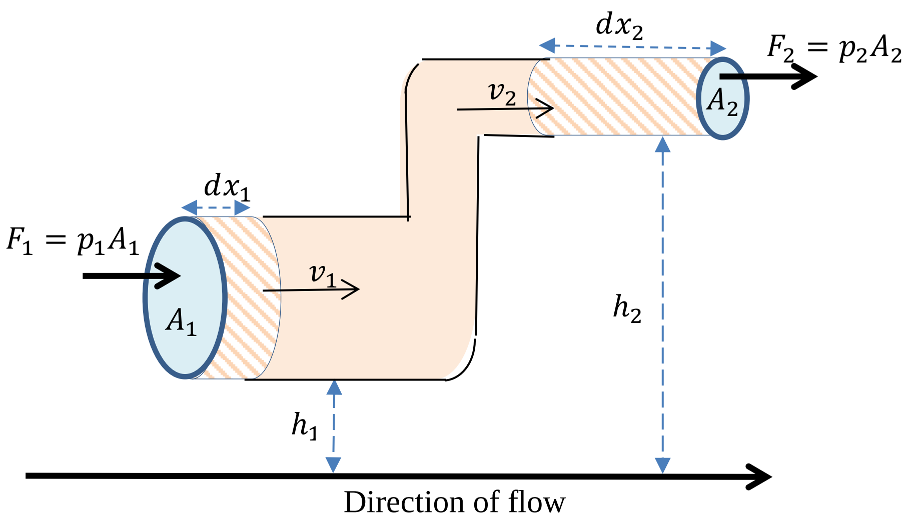
Equating (iii) and (v)

$A_1 v_1 = A_2 v_2$ (vi)

Equation (vi) represents the continuity equation. It implies that if $A_1 > A_2$ then $v_2 > v_1$. Put another way, when a liquid flows from a wider pipe to a narrower pipe, the speed of the liquid increases. This happens so as to maintain a constant volume flow rate of the liquid. Assumptions made while deriving equation (vi) include:

- Liquid is flowing steadily (no turbulence as this would lead to an increase in pressure)
- Liquid is not compressible
- Liquid is non-viscous so as to reduce frictional drag. Examples of non-viscous liquids include water, petrol, paraffin etc. An example of a viscous liquid is glycerine.

Bernoulli's principle is concerned with transport and conservation of energy that include kinetic energy, gravitational potential energy and energy on account of pressure. Consider a liquid flowing in a pipe of different cross-sectional areas and at different elevations from a reference point as shown.



The liquid possesses energy in the form of:

(i) Kinetic energy (K): Suppose in time t, a liquid of mass m and density ρ moves through a small distance dx_1 in the wide pipe with velocity v_1. When the liquid flows into the narrow pipe, the velocity increases, say to v_2, in accordance with the continuity equation, covering a distance dx_2 within time t (time t constant hence $dx_2 > dx_1$). The change in kinetic energy ΔK when the liquid moves from the wider to the narrow pipe becomes;

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}m(v_2^2 - v_1^2) \quad \text{(vii)}$$

But $m = \rho dV$ where $dV = A_1dx_1 = A_2dx_2$ is the volume of the liquid that has moved through the pipe in time t. Hence;

$$\Delta K = \frac{1}{2}\rho dV(v_2^2 - v_1^2) \quad \text{(viii)}$$

(ii) Gravitational potential energy. The liquid in the wide and narrow pipes possess potential energy by virtual of their elevation. If h_1 and h_2 be the elevations of the liquids in the wide and narrow pipes respectively, then the change in potential energy ΔU when the liquid flows from the wide to the narrow pipe is given by:

$$\Delta U = mgh_2 - mgh_1 = mg(h_2 - h_1) \quad \text{(ix)}$$

But $m = \rho dV$ hence;

$$\Delta U = g\rho dV(h_2 - h_1) \quad \text{(x)}$$

(iii) Work done by pressure: Suppose the liquid in the wide pipe is under pressure p_1. This pressure generates force F_1 considering;

$$p_1 = \frac{F_1}{A_1} \quad \text{(xi)}$$

$$F_1 = p_1A_1 \quad \text{(xii)}$$

Similarly, if p_2 be the pressure of the liquid in the narrow pipe and F_2 the force generated, then;

$$p_2 = \frac{F_2}{A_2} \quad \text{(xiii)}$$

$$F_2 = p_2 A_2 \quad \text{(xiv)}$$

The forces F_1 and F_2 drive the liquid through distances dx_1 and dx_2 respectively and therefore do work equal to the product of the force and the respective distance. The change in the work done ΔW_p on account of pressure is therefore equal to;

$$\Delta W_p = p_2 A_2 dx_2 - p_1 A_1 dx_1 \quad \text{(xv)}$$

But $A_2 dx_2 = A_1 dx_1 = dV$ hence;

$$\Delta W_p = p_2 dV - p_1 dV \quad \text{(xvii)}$$

$$\Delta W_p = (p_2 - p_1) dV \quad \text{(xviii)}$$

Given that the total energy of the system is conserved, it follows that no change in total energy occurs as the liquid moves from the wide to the narrow pipe. The total change in energy therefore equals zero, that is;

$$\Delta W + \Delta K + \Delta U = 0 \quad \text{(xix)}$$

Using equations (viii), (x) and (xviii) in equation (xix) leads to;

$$\frac{1}{2}\rho dV(v_2^2 - v_1^2) + g\rho dV(h_2 - h_1) + (p_2 - p_1)dV = 0 \quad \text{(xx)}$$

$$p_2 - p_1 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + g\rho h_2 - g\rho h_1 = 0 \quad \text{(xxi)}$$

$$p_1 + \frac{1}{2}\rho v_1^2 + g\rho h_1 = p_2 + \frac{1}{2}\rho v_2^2 + g\rho h_2 \quad \text{(xxii)}$$

In general, equation (xxii) may be expressed as;

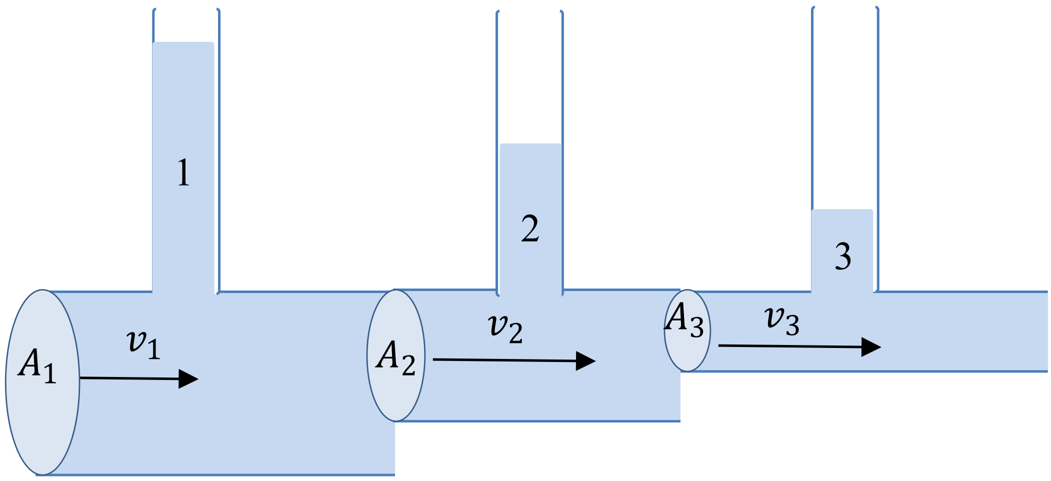
$$p + \frac{1}{2}\rho v^2 + g\rho h = constant \quad \text{(xxiii)}$$

Equation (xxiii) is a mathematical representation of Bernoulli's principle. It shows that a reduction in the speed of a fluid leads to an increase in pressure.

NOTE 1: Motion of a fluid is represented by lines called streamlines. If the streamlines are close together, speed is high and pressure is low. If far apart, speed is low hence pressure is high.

NOTE 2: Implications of Bernoulli's principle

- Consider a liquid flowing in a pipe of different cross-sectional areas A_1, A_2 and A_3 where $A_1 > A_2 > A_3$, with velocities v_1, v_2 and v_3 respectively. Suppose too that tubes 1,2 and 3 are fixed in the sections of the pipe with cross section areas A_1 A_2 and A_3 respectively. If the atmospheric pressure is lower than the liquid pressure, the higher liquid pressure pushes the liquid up the tubes with the liquid attaining the highest level in tube 1, followed by tube 2 and lastly tube 3.



According to the continuity equation

$A_1v_1 = A_2v_2 = A_3v_3$

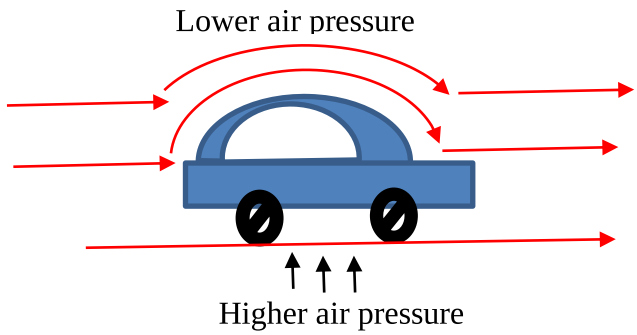
This means that since $A_1 > A_2 > A_3$ then $v_1 < v_2 < v_3$.

By Bernoulli's equation (assuming h to be negligible);

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 p_1 = p_3 + \frac{1}{2}\rho v_3^2$$

Since $v_1 < v_2 < v_3$, then $p_1 > p_2 > p_3$. Liquid pressure therefore increases with the cross-sectional area. It is for this reason that the liquid attains the highest level in tube 1.

- A speeding car appears to fly (barely touch the ground). This is because the speed of air above the car is higher than under the car leading to regions of low and high pressure respectively, with the higher pressure forcing the car upwards.



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- Rooftops are blown away during wind storms due to lower pressure over the roof compared to inside the house.
- Vegetation bends towards busy roads because speeding cars create a region of low pressure near the road. The higher pressure

on the other side of the vegetation pushes the vegetation towards the road.

- When air is blown between two papers, the papers move closer as pressure between them reduces.

To the question:

The syphon should be filled with the liquid to drive out air hence remove the effect of atmospheric pressure which would otherwise prevent water from flowing out as pressure inside the tube would be equal to pressure outside the tube.

(b) End X must be below the level of the liquid in the tank (1 mark

To create pressure difference

3. Figure 9 shows a folded piece of paper. A stream of air is blown underneath the paper.

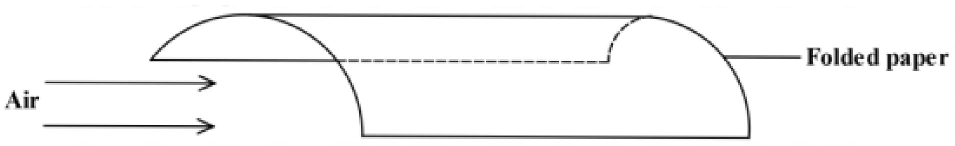


Figure 9

Explain why the paper collapsed. (2 marks)

When air is blown underneath the paper, the air molecules are forced to move faster thereby creating a region of lower pressure in accordance with Bernoulli's principle. The higher atmospheric pressure above the paper forces it downwards hence the collapse.

4. Figure 3(a) shows a horizontal tube containing air trapped by a mercury thread of length 5cm. The length of the enclosed air column is 7.5cm. The atmospheric pressure is 76cmHg.

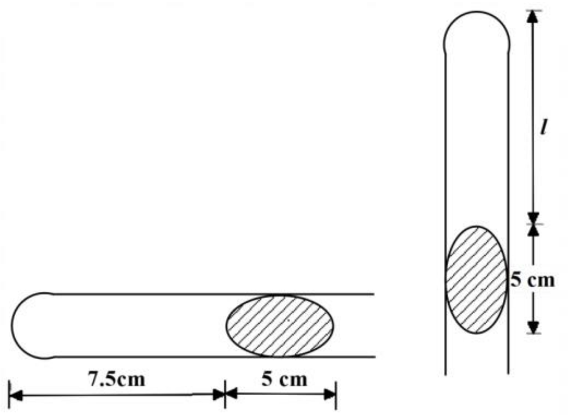


Figure 3(a) Figure 3(b)

The tube is then turned vertically with its mouth facing down as shown in Figure 3(b).
(a) Determine the length l of the air column. (3 marks)

Gas laws and the kinetic theory of gases

NOTE 1: Gas laws

When an inflated balloon is left out in the sun, it increases in volume without extra air being pumped in. If the balloon is taken out of the sun and placed in a cold room, the volume reduces without any air escaping. Out in the sun, the temperature of air in the balloon rises and the gas expands leading to an increase in volume. In the cold room, the temperature of the air in the balloon reduces and consequently the air contracts hence volume reduces. This implies that when temperature of air (gas) increases, the volume of the gas increases and vice versa. For this to happen, the pressure and mass of the gas must be kept constant. Hence

$$V \infty T \qquad \text{(i)}$$

$$\frac{V}{T} = constant \qquad \text{(ii)}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad \text{(iii)}$$

Equation (i-iii) represents Charles law which states that pressure constant, the volume of a given mass (moles) of a gas is directly proportional to the temperature of the gas.

Now, suppose that the balloon is loosely inflated. If the volume of the balloon is gradually decreased without letting out any air, the balloon becomes firmer and eventually bursts. The reason for these observations is that initially, the pressure of air inside the balloon is equal to the atmospheric pressure hence the balloon remains inflated. However, as the volume of the balloon is reduced, the trapped air particles are forced closer to each other (air is compressible). This leads to increased collisions of particles within the balloon leading to an increase in pressure. At some point, the pressure inside the balloon increases much higher than the atmospheric pressure causing the balloon to burst. Thus, when the volume of a fixed mass of gas at a constant temperature reduces, pressure increases and vice-versa, i.e.,

$$p \infty \frac{1}{V} \tag{iv}$$

$$pV = constant \tag{v}$$

$$p_1V_1 = p_2V_2 \tag{vi}$$

Equations (iv-vi) represent Boyles law and only hold if the temperature and mass (number of moles) of the gas are kept constant.

Suppose now that a rigid and empty metal can is sealed in such a way that the air inside remains trapped. When the can is gradually heated, it eventually explodes. The reason for this is that as the temperature of the air in the can increases, the kinetic energy of the air molecules increases leading to increased collisions and therefore an increase in pressure. When the air pressure exceeds the atmospheric pressure, the can eventually explodes. Thus, an increase in temperature of a fixed mass of gas, volume constant, leads to an increase in the gas pressure and vice versa. Hence;

$$p \infty T \tag{vii}$$

$$\frac{p}{T} = constant \tag{viii}$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \tag{ix}$$

Equations (vii-ix) represents pressure law and only holds when volume and mass (number of moles) of the gas are constant

The three laws, Charles's, Boyle's and pressure laws, are collectively referred to as gas laws. Real gases obey the gas laws only at very low pressure. A hypothetical gas that obeys the gas laws perfectly at all time is referred to as an ideal gas.

For a gas system containing *n* moles of an ideal gas, equations (i) (iv) and (vii) can be combined into a single equation as;

$$pV \infty T \tag{x}$$

$$\frac{pV}{T} = constant \tag{xi}$$

The constant is equal to the product of the number of moles of the gas (*n*) and a constant of proportionality *R* referred to as the universal gas constant. Equation (xi) may therefore be written as;

$pV = nRT$ (xii)

Equation (xii) is referred to as the ideal gas equation. If for example the state of a gas system of *n* moles changes from $(p_1V_1T_1)$ to $(p_2V_2T_2)$ then by equation (xii), it follows that;

$\frac{p_1V_1}{T_1} = nR$ (xiii)

$\frac{p_2V_2}{T_2} = nR$ (xiv)

The right-hand side of equation (xiii) equals the right-hand side of equation (xiv) hence;

$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ (xv)

NOTE 2: Gas laws verification

According to Charles law, volume *V* of a fixed mass of gas is directly proportional to the temperature *T*, pressure constant.

$V \infty T$ (i)

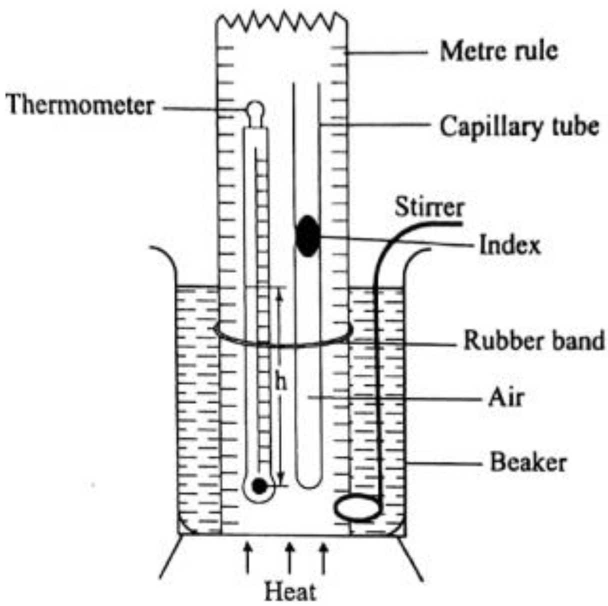
For a column of air of length *l* and cross-sectional area *A*;

$V = Al$ (ii)

If the cross-sectional area is kept constant, Charles law can be expressed as;

$l \infty T$ (iii)

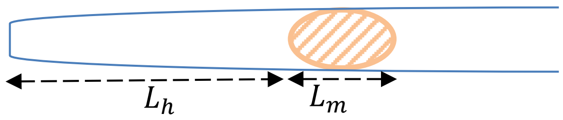
To prove Charles' law, air is trapped in a test-tube using a moveable index. The test-tube is dipped in cold water and after the index has stabilized the temperature of the trapped air T_i (which is equal to the temperature water) and the corresponding position of the index L_i obtained.



The water is gradually heated while being continuously stirred and the temperature of air in the tube, *T* and the corresponding position of the index *l* obtained periodically. A graph of *l* against *T* is then plotted. A straight line inclined to the horizontal proves Charles' law.
Important points to note;

- To keep the pressure constant, the test tube is kept open and the index fixed in such a way that it is free to move up and down without letting trapped air out.
- The length of the air column trapped in the test-tube is used in place of volume since the cross-sectional area is constant.
- Water is continuously stirred to distribute heat evenly to ensure that the temperature of air is accurately measured.

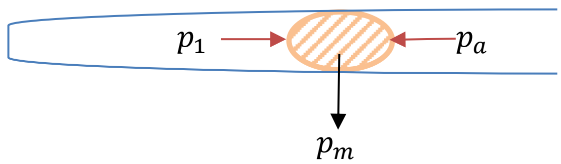
(ii) Boyle's law:
An air column is trapped in a test tube of cross-sectional area *A* by a mercury thread of length L_m that is free to move about without letting air in or out of the tube. The tube is placed horizontally and the length of the air column L_h measured.



If V_1 be the volume of the trapped air, then

$V_1 = AL_h$ (i)

The mercury thread is not resting on the trapped air column hence does not exert pressure (p_m) on the air. The atmosphere on the other hand exerts pressure p_a in all directions. Considering that the system is at equilibrium, it follows that the pressure p_1 the trapped air exerts is equal to the atmospheric pressure.



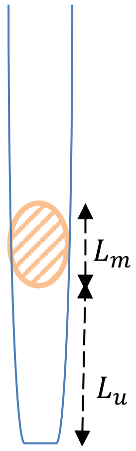
That is;

$p_1 = p_a$ (ii)

Multiplying equations (i) and (ii) leads to;

$p_1V_1 = p_aAL_h$ (iii)

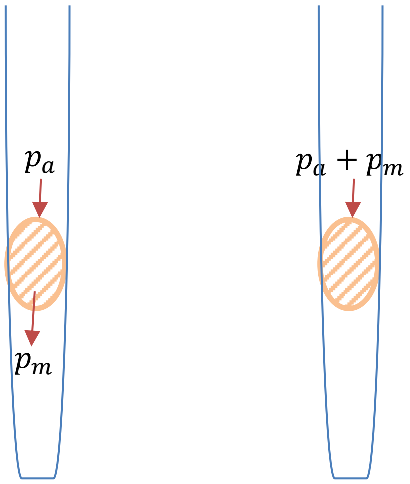
The tube is now placed in an upright position and the length L_u of the air column measured (the length of the air column should be observed to decrease, that is, $L_u < L_h$).



If V_2 be the volume of the tapped air, then;

$V_2 = AL_u$ (iv)

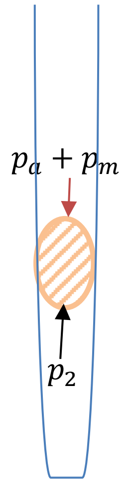
The mercury thread this time rests on the trapped air column hence exerts pressure (p_m) on it. The atmosphere too exerts pressure p_a on the air column.



The resultant pressure p_{am} on the trapped air is therefore equal to;

$p_{am} = p_a + p_m$ (v)

It is this increased pressure that leads to a reduction in length of the trapped air column. Since the system is in equilibrium, the pressure the mercury column and the atmosphere exert on the trapped air should be equal to the pressure p_2 the gas exerts in the opposite direction.



That is;

$p_2 = p_a + p_m$ (vi)

Multiplying ((i) by (vi) leads to;

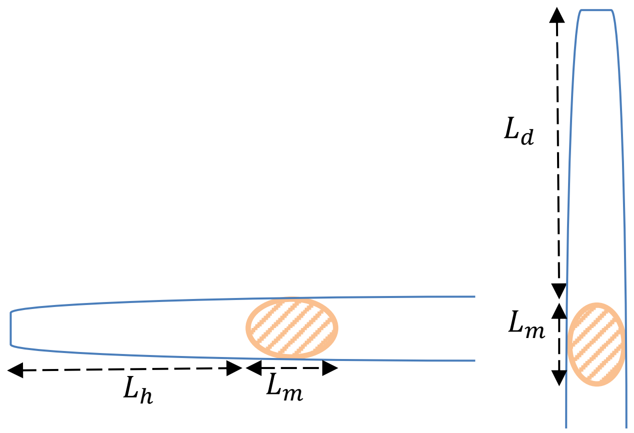
$p_2V_2 = (p_a + p_m)AL_u$ (vii)

If the trapped air obeys Boyle's law, then equations (iii) and (vii) should be equivalent, that is;

$p_1V_1 = p_2V_2$

$p_aL_h = (p_a + p_m)L_u$ (viii)

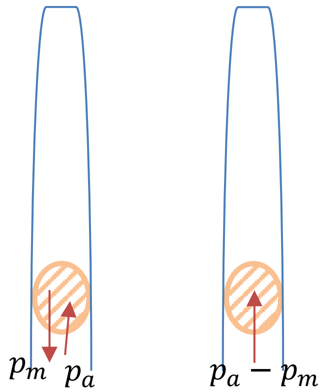
Suppose now the tube is turned upside down.



The length of the air column, say L_d should be observed to increase, that is, $L_d > L_h$). If V_2 be the volume of the tapped air, then;

$V_2 = AL_d$ (ix)

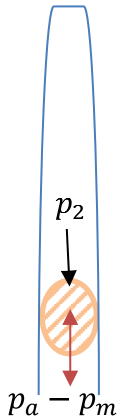
The mercury thread does not exert pressure on the trapped air. However, the thread exerts pressure p_m on the atmosphere. This counters the pressure p_a the atmosphere exerts on the trapped gas (p_m and p_a are in opposite direction).



The resultant pressure p_{am} on the trapped air is therefore equal to;

$p_{am} = p_a - p_m$ (x)

The reduced resultant pressure on the trapped air is responsible for the longer column. If p_2 be the pressure the trapped air exerts, then at equilibrium this pressure should be equal to the resultant pressure due to the atmosphere and the mercury column.



$p_2 = p_a - p_m$ (x)

Multiplying ((ix) by (x) leads to;

$p_2V_2 = (p_a - p_m)AL_d$ (xi)

If the air obeys Boyle's law, then equations (iii) and (xi) should be equivalent, that is;

$p_1V_1 = p_2V_2$

$p_aL_h = (p_a - p_m)L_d$ (xi)

NOTE 3: Kinetic theory of gases

According to the kinetic theory of gases, gas molecules with temperature above the absolute zero (0 K) are in constant random motion (called Brownian motion), colliding with each other and with the walls of the container they are placed in. These collisions account for gas pressure. If for example the temperature of a fixed mass of gas is increased, the kinetic energy of the particles increases and to keep pressure constant, the particles drift further apart leading to an increase in volume. This accounts for Charles law. If on the other hand the gas is restricted to a rigid container (constant volume), an increase in kinetic energy causes the gas molecules to collide faster leading to an increase in gas pressure in accordance with the pressure law. If the temperature is kept constant and the volume of the gas reduced (gases are compressible), the gas particles move closer to each other hence the collisions and consequently the gas pressure increases which is in line with Boyle's law. The random movement of molecules of a gas above 0 K is the basis of the kinetic theory of gases. A number of assumptions are however made in relation to the kinetic theory of gases;

- Molecules of a given gas are identical
- Collisions between particles and the container are perfectly elastic and therefore energy and momentum are conserved.
- Molecules do not exert any force on other molecules except during collisions. The influence of gravity on the particles is also ignored.
- The number of particles is high enough for statistics to be meaningfully applied.
- The size of molecules is negligible compared to their separation.
- The laws of Newtonian (classical) mechanics apply (as opposed to quantum mechanics).

To the question

$$p_a L_h = (p_a - p_m) L_d$$

$$76 \times 7.5 = (76 - 5) L_d$$

$$L_d = \frac{76 \times 7.5}{71} = 8.028\, m$$

(b) State the reason why the mercury thread did not fall out in Figure 3(b).

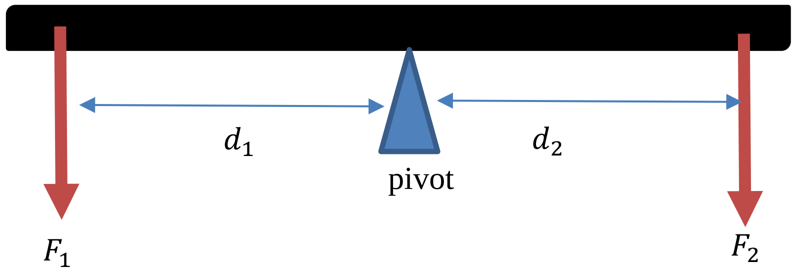
The upward atmospheric pressure is greater than the downward pressure exerted by the trapped air and the mercury thread.

5. State the reason why a student climbing a hill tends to bend forward. (1 mark)

Moment of a force is defined as the product of force and the perpendicular distance between the force and the point of support (pivot, fulcrum). Equilibrium (balance) occurs when:

$$Anticlockwise\ moment = clockwise\ moment \qquad \text{(i)}$$

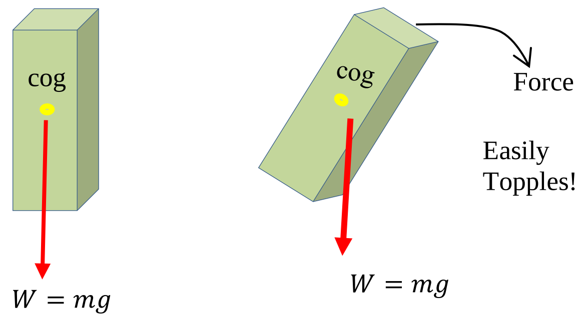
From the diagram below, at equilibrium:



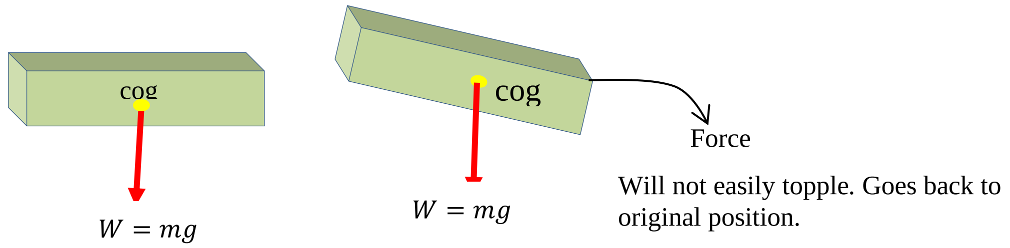
$F_1 d_1 = F_2 d_2$ (ii)

Center of gravity (cog) is the point on a body where the entire weight of the body acts. The lower the center of gravity, the more stable an object is. This means that it is more difficult to topple a stable object compared to an unstable one. One way of lowering the center of gravity of an object is by making the base relatively heavy. Buses for example are constructed with luggage cabins on the lower sections, with limited load (carry-on luggage) being allowed in.

Consider a block of wood with the geometrical center as cog resting on its smaller surface. When a small amount of force is applied, the vertical line through the cog falls outside the base of the block and it topples over.



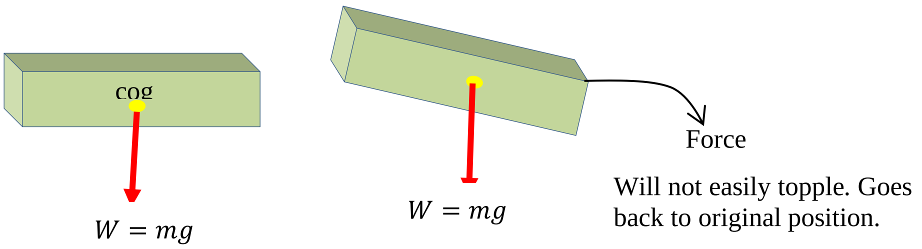
When the block rests on the wider side, some small force will not topple the block since the vertical line through the cog still passes through its base.



There are three types of equilibrium

(i) Stable equilibrium

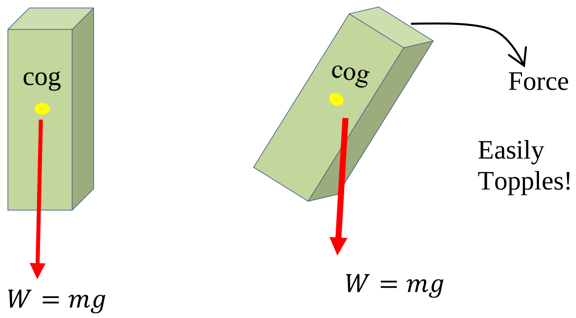
This occurs when a body displaced from the equilibrium position falls back to its initial position. Net torque (turning effect) of the body is opposite the direction of the applied force. Example;



Relatively short objects with heavy bases and large cross-sectional areas are associated with stable equilibrium.

(ii) Unstable equilibrium

This occurs when a body displaced from the position topples over when a small force is applied. Net torque (turning effect) of the body is in the same direction as the applied force. Example;



Relatively tall objects with small bases and high center of gravity are associated with unstable equilibrium.

(iii) Neutral equilibrium

This occurs when the equilibrium of a body is independent of the displacement from its initial position. Example a ball.

NOTE 1: A student carrying a heavy box using the right hand is observed to lean towards the left-hand side. This is because the box shifts the position of the center of gravity of the system towards the right hand and consequently to balance moments and hence maintain balance, the student leans in the opposite direction. If the center of gravity shifts, a force must be exerted in a direction opposite to that of the shift to maintain equilibrium.

NOTE 2: Conditions necessary for a body to be in equilibrium:

- Clockwise moments must be equal to anticlockwise moments
- The resultant force on the body should be zero. This means that if all the forces acting on a body were resolved into vertical (up and down) and horizontal (left and right) components, then the up-forces should be equal to the down-forces, and the left-forces equal to the right-forces.

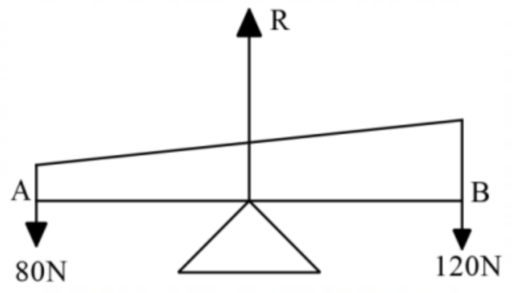
NOTE 3: A student climbing a hill tends to bend forward so as to shift the position of the centre of gravity to the front part to maintain equilibrium while one going down the hill tends to lean backward so as to shift the center of gravity to the backside in order to maintain balance.

To the question:
A student climbing a hill tends to bend forward so as to shift the position of the centre of gravity to the front part to maintain equilibrium.

6. a) State two conditions necessary for a body to be in equilibrium. (2 marks)

- Clockwise moments must be equal to anticlockwise moments
- The resultant force on the body should be zero.

b) Figure 13 shows a non-uniform log of wood AB of length 4m. The log is held horizontally by applying forces of 80N at end A and 120N at end B.



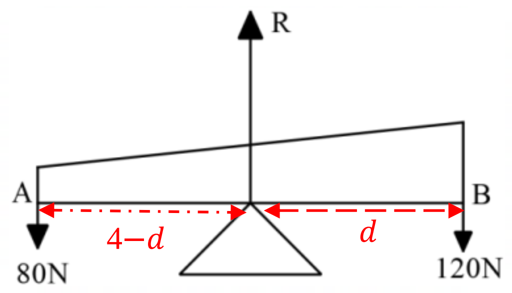
Determine:
(i) the value of R. (1 mark)

$Upward\ forces = Downward\ forces$

$R = 80 + 120 = 200\ N$

(ii) the position of the centre of gravity of the log from end B. (3 marks)

Since the log is balanced, the center of gravity coincides with the position of the pivot. If d be the distance between the pivot and B then the distance between pivot and A equals $4 - d$.



$$clockwise\ moments = anticlockwise\ moments$$
$$180d = 80(4 - d)$$
$$180d = 320 - 80d$$
$$180d - 80d = 320$$
$$d = \frac{320}{100}$$
$$d = 3.2\ m$$

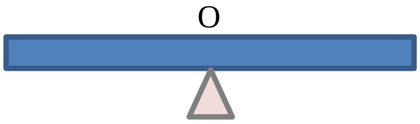
c) You are provided with the metre rule, a knife edge and a mass m_1.

(i) Describe how the position of the centre of gravity of the metre rule can be determined using the knife edge. (2 marks)

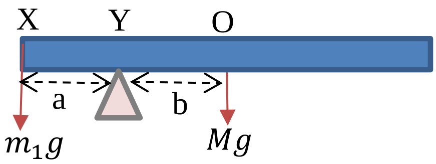
An object balances when pivoted at its center of gravity. To determine the cog of the meter rule, the meter rule is placed on the knife edge. The location of the meter rule is then varied until it balances. The point of the knife edge corresponds with the cog of the rule.

(ii) Using the position of centre of gravity determined in 19(c)(i) and the mass m_1, describe how the mass M of the metre rule can be determined. (4 marks)

Note the position, say O, of the pivot on the meter rule. This is the point where the weight Mg of the meter rule acts.



Hang the mass m_1 to the left of O, say at point X. Move the pivot between points O and X until the meter rule balances (say at point Y). Determine the distance between X and Y (say *a*) and that between Y and O (say *b*).



Then use the equation

$$m_1ga = Mgb$$
$$M = \frac{m_1a}{b}$$

7. Figure 7 shows a uniform rod AB 2m long and of mass 1 kg. It is pivoted 0.5m from end A and balanced horizontally by a string attached 0.1m from end B.

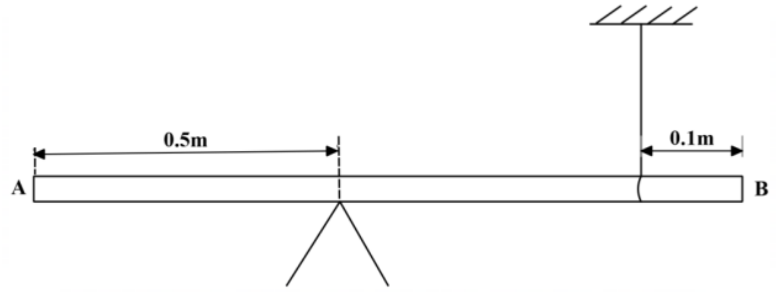
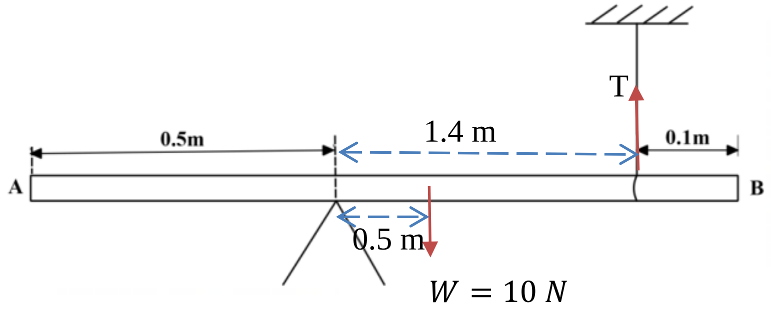


Figure 7

Determine the tension in the string. ($take\ g$ = 10Nkg^{-1}) (2 marks)

The weight W of the uniform bar acts downwards at its geometrical center O, that is 1 m from end A. distance of this point O from the pivot equals $1 - 0.5 = 0.5\ m$. The tension T acts upwards at a distance $2 - (0.1 + 0.5) = 1.4\ m$ from the pivot



$Sum\ of\ clockwise\ moments = sum\ of\ anti-clockwise\ moments$

$10 \times 0.5 = 1.4 \times T$

T = 3.57N

8. Figure 12 shows hydraulic lift system. The radius of the small piston is 5.64cm while that of the larger piston is 14.24cm. The small piston is operated using a lever. A force of 100N is applied to the lever.

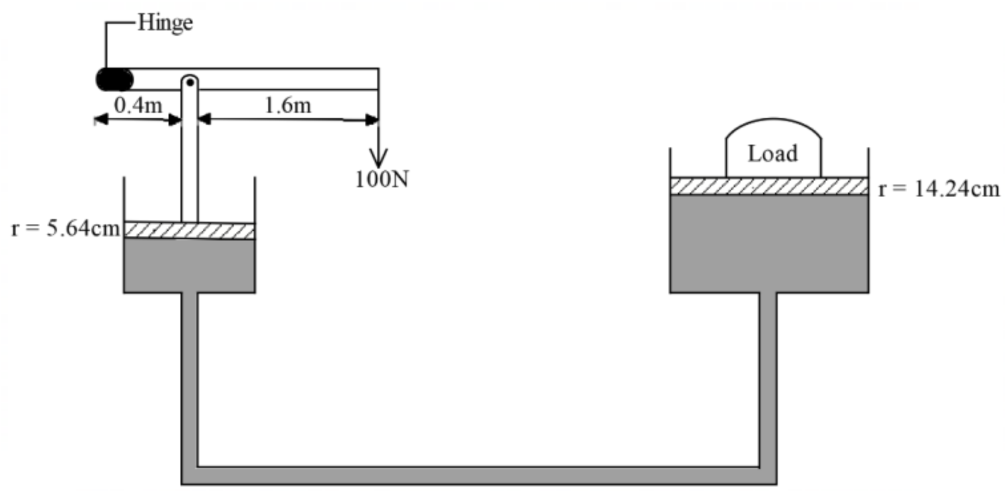
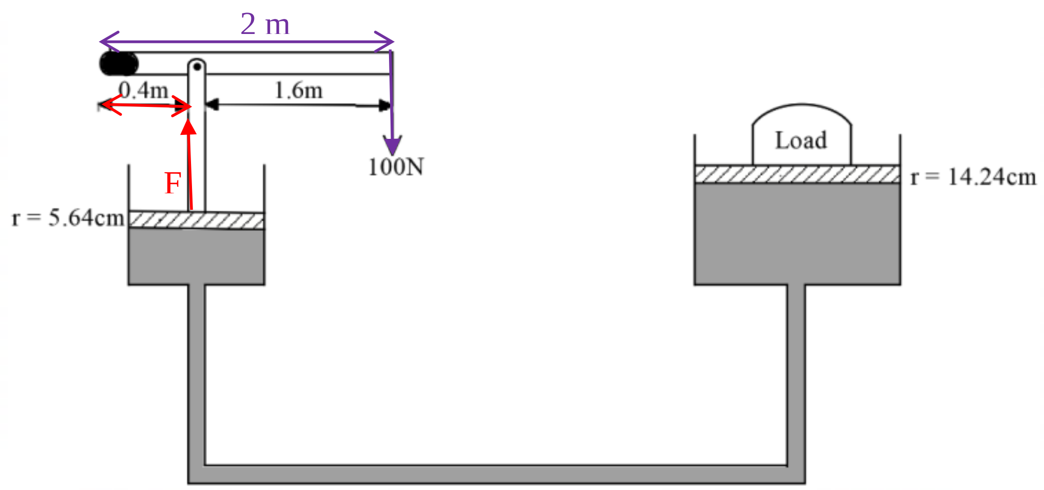


Figure 12

Determine the: (a) pressure exerted by the smaller piston. (5 marks)

$$Pressure\ (p) = \frac{Force\ (F)}{Area\ (A)}$$



The force F on the lever arm produces anticlockwise moments about the hinge while the effort (= 100 N) produces clockwise moments about the same hinge.

Clockwise moments = anticlockwise moments

$100 \times 2 = F \times 0.4$

$$F = \frac{200}{0.4} = 500\ N$$

Area of the small piston $A = \pi r^2 = 3.14 \times 0.0564^2 = 0.01\ m^2$

$$p = \frac{F}{A} = \frac{500}{0.01} = 50{,}000\ Pa$$

b) load that can be lifted. (3 marks)

Pressure generated at the smaller piston is distributed evenly to the larger piston.

$$p = \frac{Load\ (L),}{area\ (A)}$$

$L = pA$

$p = 50{,}000\ Pa$

Area of the larger piston $A = \pi r^2 = 3.14 \times 0.1424^2$

$L = 50{,}000\ \times 3.14 \times 0.1424^2$

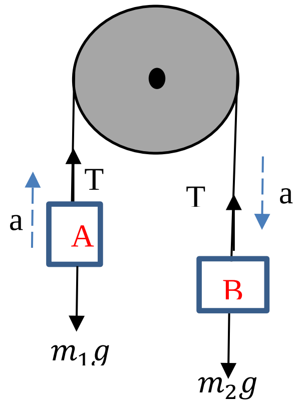
$L =\ 3183.6\ N$

(c) mechanical advantage of the system. (3 marks

Pulleys and levers

Pulleys are mechanisms composed of a wheel (or multiple wheels) with a rope threaded over the wheels. One end of the rope supports the load while the other supports the effort. Pulleys are used to lift heavy objects for example at a construction site.

The simplest pulley system is the Atwood's machine, a single pulley system with two objects attached on either side of the pulley. The objects can be of equal or unequal masses. Consider a frictionless Atwood's pulley system consisting of two objects A and B of masses m_1 and m_2 respectively. Suppose B is moving downwards thereby pulling A upwards. Both masses have the same acceleration, say *a,* but in different directions. Say *T* be the tension in the rope.



Since mass A is accelerating upwards, the tension is greater than weight hence by Newton's second law of motion:

$T - m_1 g = m_1 a$ (i)

Mass B is accelerating downwards hence its weight is greater than tension. Thus;

$m_2 g - T = m_2 a$ (ii)

Both tension T and acceleration a can be obtained using equations (i) and (ii) if other values are given.

A block and tackle pulley system consists of two or more pulleys with a rope or cable threaded between them. It is assembled in paired-pulley blocks with one pulley in one of the pairs fixed while the other one moves with the load.

Terms associated with pulleys

- Effort: The force applied to overcome the load
- Load: The force being overcome
- Mechanical advantage (MA): This is defined as the ratio of the load (L) to the effort (E), that is:

  $MA = \frac{L}{E}$ (i)

- Velocity ratio (VR): This is the ratio of distance travelled by the effort (d_E) to distance travelled by load (d_L);

  $VR = \frac{d_E}{d_L}$ (ii)

  Velocity ratio is also equal to the number of ropes supporting the load (number of times the same rope runs through the moving block). Additionally, if the pulley system is used in such a way that the effort pulls downwards while the load moves upwards, velocity ratio is equal to the total number of pulleys in the system.

- Work input (W_{in}): This is the work done by the effort. It is the product of effort and effort distance, i.e.

  $W_{in} = Ed_E$ (iii)

- Work output (W_{out}): This is the work done on the load. It is the product of load and load distance, i.e.,

  $W_{out} = Ld_L$ (iv)

- Efficiency: In reality, work input is always greater than work output because some of the work input is used to overcome friction and converted to heat, or is converted to sound. The ratio of work output to work input, often expressed as a percentage, is referred to as efficiency, i.e.

  $Efficiency = \frac{Work\ output}{Work\ input} \times 100\%$ (v)

  By equations (iii) and (iv) equation (v) may be expressed as;

  $Efficiency = \frac{Ld_L}{Ed_E} \times 100\%$ (vi)

  $Efficiency = \frac{L}{E} \times \frac{d_L}{E} \times 100\%$ (vii)

But $\frac{L}{E} = MA$ and $\frac{d_L}{d_E} = \frac{1}{VR}$ hence;

$$Efficiency = \frac{MA}{VR} \times 100\% \qquad \text{(viii)}$$

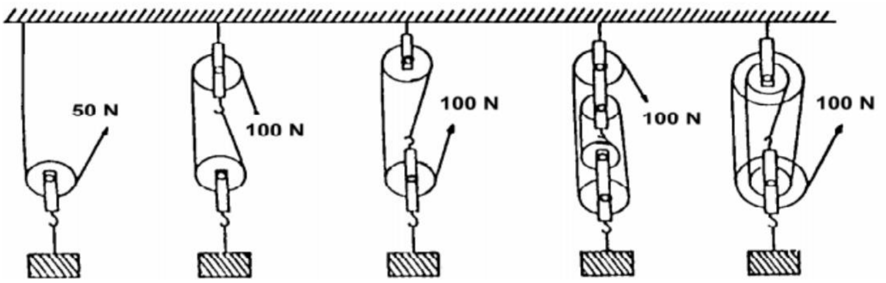
For an ideal pulley system where no energy is lost, the efficiency is 100%. In this case,

$$MA = VR = \frac{Load}{Effort} \qquad \text{(ix)}$$

The maximum load that a pulley system that is 100% efficient can lift is therefore given as;

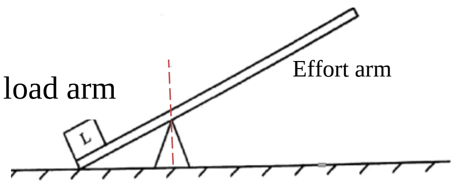
$$Load = VR \times Effort \qquad \text{(x)}$$

Example, pulleys given below are 100% efficient. Then;



$VR = 2$	$VR = 2$	$VR = 3$	$VR = 4$	$VR = 5$
$L = 2 \times 50$	$L = 2 \times 100$	$L = 3 \times 100$	$L = 4 \times 100$	$L = 5 \times 100$
$L = 100\ N$	$L = 200\ N$	$L = 300\ N$	$L = 400\ N$	$L = 500\ N$

A lever is basically a plank of wood pivoted somewhere in the middle with the effort arm (where the effort is applied) longer than the load arm (where the load is placed).



The mechanical advantage (MA) of the lever is

$$MA = \frac{load}{effort}$$

Velocity ratio is defined as the ratio of the effort distance to the load distance or ratio of effort arm length to load arm length;

$$VR = \frac{effort\ dist}{load\ dist} = \frac{effort\ arm\ length}{load\ arm\ length}$$

Efficiency is given by:

$$Efficiency = \frac{MA}{VR} \times 100\%$$

To the question:

$$MA = \frac{load}{effort} = \frac{3183.6}{500} = 6.307$$

9. a) Figure 10 shows a wooden block of volume 90cm^3 floating with a third of its body submerged in water of density 1gcm^{-3} . (g = 10Nkg^{-1})

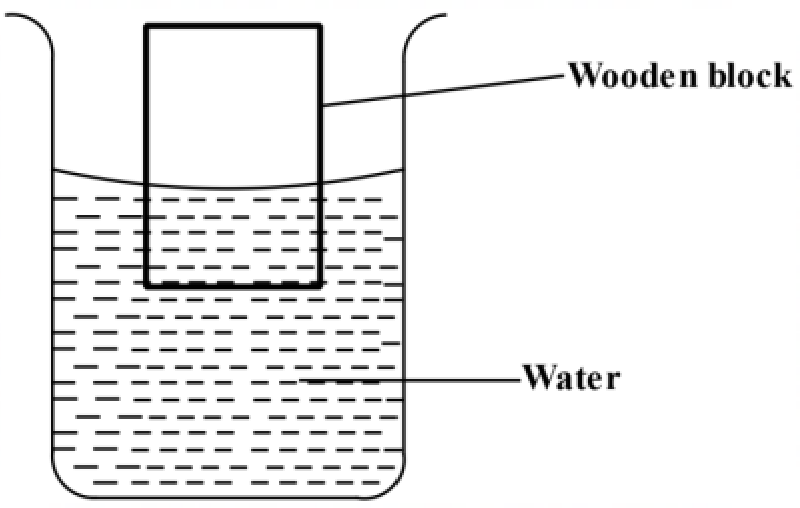
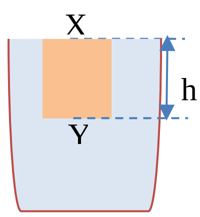


Figure 10

Determine:

(i) the weight of the block

Objects float if they are less dense than, or of equal density with, the fluids (liquids and gases) they occupy. For example, while steel is denser than water, a steel ship can be made to float by introducing air-filled chambers which reduce the density of the ship relative to that of water. Floating bodies usually have part (or all) of their bodies in the liquid and as a result experience an upward force called upthrust (or buoyant force). Consider for instance an object of mass *m*, cross-sectional area *A* and height *h* fully submerged in a liquid of density *ρ*.



The upward pressure the liquid exerts on the object at X is less than pressure the liquid exerts on the object at Y which is equal to ρgh. Also, the pressure the object exerts downwards on the liquid at X is less than that it exerts on the liquid at Y which is equal to mg/A. For the body not to sink below point Y, the upward force and hence pressure must be equal to the downward force (hence pressure) at that point. Thus,

$\rho gh = mg/A$ (i)

$\rho g(hA) = mg$ (ii)

But $hA = V$ (iii)

Where *V* is the volume of liquid displaced which is equal to the volume of the submerged part of the object. Hence;

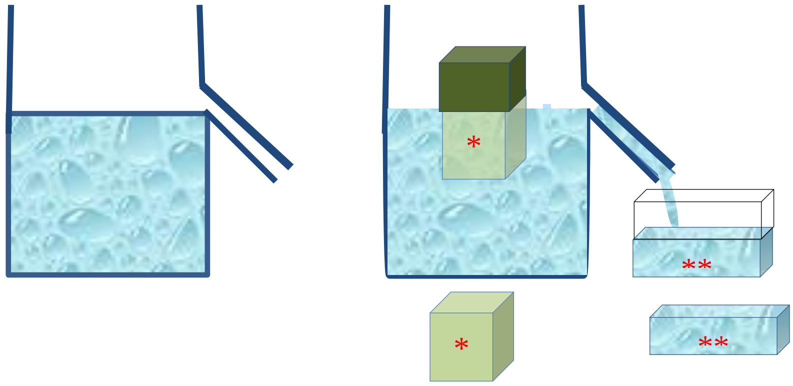
$\rho Vg = mg$ (iv)

Now, $\rho V = M\ (mass\ of\ the\ liquid\ displaced)$ (v)

So;

$Mg = mg$ (vi)

Equation (vi) shows that when an object floats in a liquid (or fluid in general), the weight of the liquid displaced is equal to the weight of the submerged section of the object. This is known as the law of floatation.



Law of floatation: $Weight\ of\ * =\ weight\ of\ **$

The terms on the left-hand sides of equations (v) and (vi) represent upthrust (*U*) which is the upward force the liquid exerts on the floating object, while the right-hand side represents the weight of the part of the object submerged, that is;

$\boldsymbol{U = \rho gV = Mg} = mg$ (vii)

It follows from equation (vii) that;

(i) If upthrust is greater than the weight of an object, the object floats with part of it partially submerged

(ii) If upthrust is equal to the weight of the object, the object floats while fully submerged but close to the surface.

(iii) If upthrust is less than the weight of the object, the object sinks.

(iv) The upthrust experienced by a floating object is proportional to the density of the liquid it is immersed in. It is for this reason that it is easier to float in the ocean (salty) than in a lake (fresh) as salty water is denser than fresh water.

(v) The upthrust experienced by a floating object is also proportional to the volume of the section of the object submerged. This means that the larger the body submerged, the greater the upthrust

To the question:

weight of the body = weight of the liquid displaced.

Given that

$density = \frac{mass}{volume}$

then

mass of liquid displaced; $mass = density \times vlume$

volume (V) of the liquid displaced=volume of submerged section;

$V = \frac{1}{3} \times 90 = 30\ cm^3$

Mass of liquid displaced;

$mass == 1 \times 30 = 30\ g = 0.03\ kg$

weight of the body = weight of the liquid displaced $= 0.03 \times 10 = 0.3\ N$

(ii) the weight of a metal block that can be placed onto the block so that its top surface is on the same level as the water surface. (3marks)

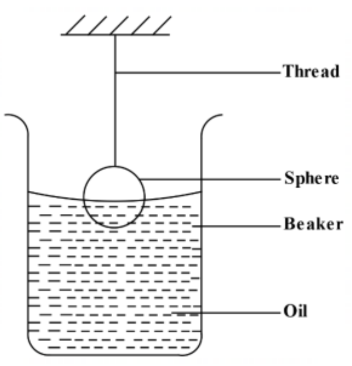
If fully submerged, volume of liquid displaced = volume of the block= $90\ cm^3$

Mass of the liquid displaced $m = V\rho = 90 \times 1 = 90\ g = 0.09\ kg$

Weight object=weight of liquid displaced $W = 0.09 \times 10 = 0.9\ N$

extra weight $= 0.9 - 0.3 = 0.6\ N$

b) Figure 11 shows a solid metal suspended in oil using a thread.



(i) Other than upthrust, list two other forces acting on the sphere. (2 marks).

-Tension

-Weight

(ii) The oil is carefully and gradually drawn from the beaker. State the effect on each of the two forces in 15(b)(i). (2 marks)

The body is under 3 forces; tension and upthrust upwards and weight downwards. As the oil is drawn from the beaker, the section of the body submerged reduces hence upthrust reduces.

- Tension gradually increases.
- Weight remains constant since it is a function of mass and acceleration due to gravity, both of which remain unchanged.

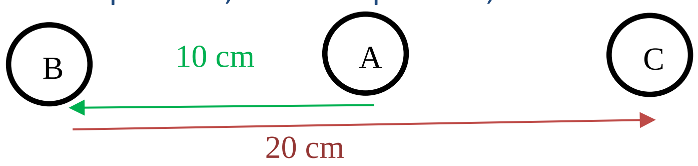
10. a) A bus moving initially at a velocity of 20 ms^{-1} decelerates uniformly at 2 ms^{-2}.
(i) Determine the time taken for the bus to come to a stop. (3 marks)

When an object moves along a straight line, it is said to undergo linear motion. If it moves along a circular path or a bend, it is said to undergo circular or rotational motion. If the object moves to-and-fro about a mean position, it is said to undergo vibratory or oscillatory motion.

Linear motion

There are various terms associated with linear motion:

Displacement (s): This refers to the distance covered by an object in a given direction. Displacement is a vector quantity hence has both direction and magnitude (distance has magnitude only). For example, suppose an object starts at point A, moves to point B, a distance of 10 m left of point A, then to point C, a distance of 20 cm right of B.



To obtain the total displacement, the direction has to be put into consideration; If the displacement to the left is considered as negative, the opposite direction (right) is taken as positive. The total displacement (s) becomes;

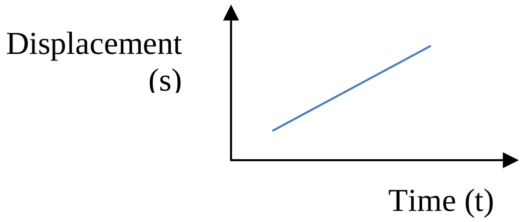
$s = (-10) + 20 = 10\ cm$

The total displacement is 10 cm to the right of A (since answer is positive).

To evaluate the distance. the direction is disregarded hence total distance d is given as:

$d = 10 + 20 = 30\ cm$

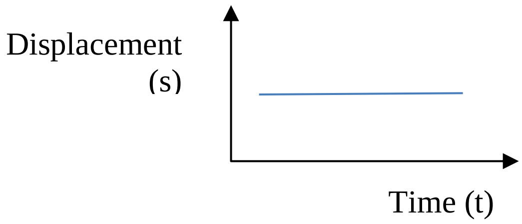
Velocity (v): This refers to the rate of change of displacement with time. It is a vector quantity with the same direction as the displacement. A body is said to move at a constant velocity if it covers equal distances within equal intervals of time. A graph of displacement against time in this case is a straight line inclined to the horizontal.



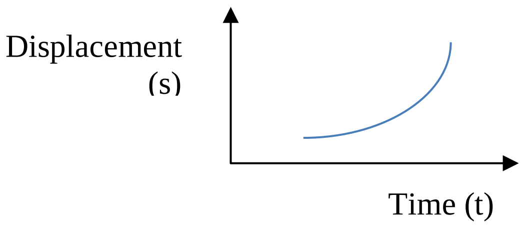
Suppose the displacement of a body changes uniformly by Δs in time Δt. The velocity v of the body becomes:

$$v = \frac{\Delta s}{\Delta t} \quad \text{(i)}$$

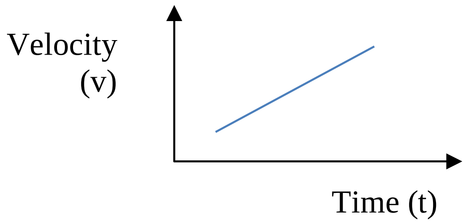
If a body is stationary (not moving), the graph of displacement against time is a straight horizontal line.



Acceleration (a): Suppose displacement of a body is not uniform over equal intervals of time.



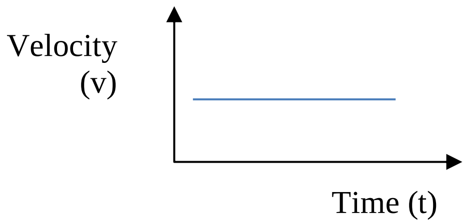
In this case, the body is said to be accelerating. Acceleration is therefore the rate of change of velocity with time. If the change in velocity is equal in equal intervals of time, the acceleration is said to be uniform and a graph of velocity against time is a straight line inclined to the horizontal.



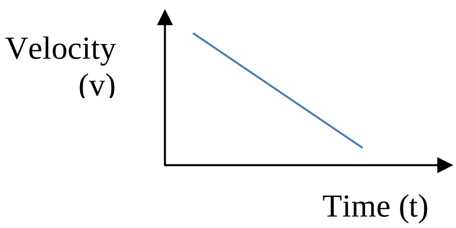
If a body initially moving with a velocity *u* (initial velocity) accelerates uniformly to a velocity *v* (final velocity) in time *t*, the acceleration *a* of the body is given by;

$$a = \frac{v-u}{t} \qquad \text{(ii)}$$

If the body is not accelerating (i.e., body moving at a constant velocity, velocity not changing with time) the velocity-time graph is a horizontal line.

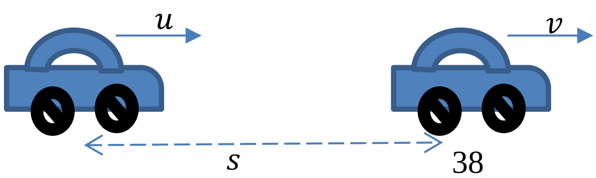


If the velocity of the body reduces with time, for example when a vehicle slows down to a stop, it is said to be decelerating or with a negative acceleration given that the initial velocity is greater than the final velocity. A graph of velocity against time in this case is a straight lined inclined to the vertical.



Equations of linear motion

The relationship between velocity, displacement and acceleration of a body undergoing linear motion are represented by three main equations referred to as equations of linear motion. Consider a car initially moving with a velocity *u*. Suppose too that in time *t*, the velocity of the car increases uniformly from *u* to *v* during which time it undergoes a displacement *s*.



It follows that the acceleration a is equal to;

$a = \frac{v-u}{t}$

$\Longrightarrow v = u + at$ (iii)

Equation (iii) is the first equation of linear motion.

Now, the average velocity v_{av} of the car is equal to;

$v_{av} = \frac{u+v}{2}$ (iv)

The displacement s of the car in time t is the product of average velocity and time, i.e.,

$s = (\frac{u+v}{2})t$ (v)

Now, equation (iii) may be expressed as;

$t = \frac{v-u}{a}$ (vi)

Using equation (vi) in equation (v) leads to;

$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right) = \frac{(v-u)(v+u)}{2a}$ (vii)

$s = \frac{v^2-u^2}{2a}$ (viii)

$v^2 = u^2 + 2as$ (ix)

Equation (ix) represents the second equation of linear motion.

Using equation (iii) in equation (v) leads to;

$s = \left(\frac{u+u+at}{2}\right)t = \left(\frac{2u+at}{2}\right)t$ (x)

$s = (u + \frac{1}{2}at)t$ (xi)

$\Longrightarrow s = ut + \frac{1}{2}at^2$ (xii)

Equation (xii) represents the third equation of linear motion.

In a nutshell, the three equations of linear motion are;

$v = u + at$

$v^2 = u^2 + 2as$

$$s = ut + \frac{1}{2}at^2$$

NOTE 1: The area under a velocity-time graph is equal to the total displacement of a body moving with constant acceleration, and equal to the total distance if the acceleration changes with time.

NOTE 2: For objects under free fall (influence of gravity only) for example when a stone is thrown up or dropped from a higher elevation, the acceleration due to gravity (g) is used. The value of g is negative when the body is moving upwards (decelerating) and positive when the body is moving downwards (accelerating).

NOTE 3: Suppose a body is launched straight up with a velocity u. Say the body reaches a maximum height s in time t. At maximum height therefore, the final velocity v equals zero. It follows that by the first equation of linear motion:

$0 = u - gt$ (xii)

$\Rightarrow u = gt$ (xiii)

$\Rightarrow t = \frac{u}{g}$ (xiv)

Equation (xiv) represents time taken by the body to reach maximum height.

Since the velocity at maximum height is zero, then by the second equation of linear motion;

$0 = u^2 - 2gs$ (xv)

$u^2 = 2gs$ (xvi)

$s = \frac{u^2}{2g}$ (xvii)

Equation (xvii) represents the maximum height reached by the body.

If the body goes up then back to its launching point, the total displacement is zero and therefore by the third equation of linear motion it follows that;

$0 = ut - \frac{1}{2}gt^2$ (xviii)

$ut = \frac{1}{2}gt^2$ (xix)

$2u = gt$ (xx)

$t = \frac{2u}{g}$ (xxi)

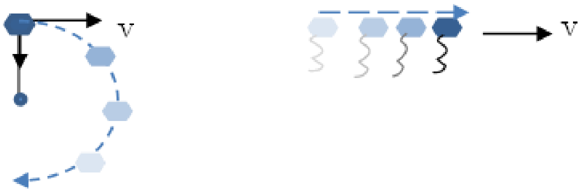
Equation (xxi) represents time of flight. The equation shows that the time of flight is twice the time taken to reach the maximum height.

Equations of motion represents the kinematics of motion, showcasing relationships between displacement, velocity, and acceleration. Objects at rest do not spontaneously move, neither do they spontaneously accelerate or decelerate if in motion. An external force must interact with the body for change of state of motion to occur. Newtons laws of motion looks at the dynamics of an object with respect to the forces acting on it and the subsequent consequences. The laws therefore describe the kinetics of an object.

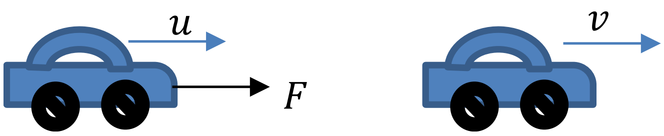
Newton's laws of motion

There are three Newton's laws of motion

(i) Newton's first law: Also referred to as the law of inertia (inertia is the tendency of a body to resist change). It states that; a body remains at rest or in motion with constant velocity along a straight line unless acted upon by an external force. For example, say a stone tied to a string is swirled around in a circular path with a linear velocity *v* (tangential to the circular path). The tension in the string keeps the stone in the circular path and if the string snaps, the stone flies straight away with a constant velocity.



(ii) Newton's second Law: This states that the rate of change of momentum is directly proportional to the force applied and takes place in the direction of the force. Consider a toy car, mass *m*, initially moving with a velocity *u*. Suppose too that a force *F* is applied on the toy car for a timeΔt during which time its velocity increases uniformly from *u* to *v.*



Considering that momentum is the product of mass and velocity, the rate of change of momentum with time, $\frac{\Delta P}{\Delta t}$, of the toy car is given as;

$$\frac{\Delta P}{\Delta t} = \frac{mv - mu}{\Delta t} \qquad \text{(i)}$$

By Newton's second law;

$$F \propto \frac{\Delta P}{\Delta t} \qquad \text{(ii)}$$

$$F \propto \frac{mv - mu}{\Delta t} \qquad \text{(iii)}$$

$$F = k\frac{mv - mu}{\Delta t} \qquad \text{(iv)}$$

where k is a constant of proportionality

$F = km\frac{(v-u)}{\Delta t} = km\frac{\Delta v}{\Delta t}$ (v)

Now, $\frac{\Delta v}{\Delta t} = a(aceleration)$ hence

$F = kma$ (vi)

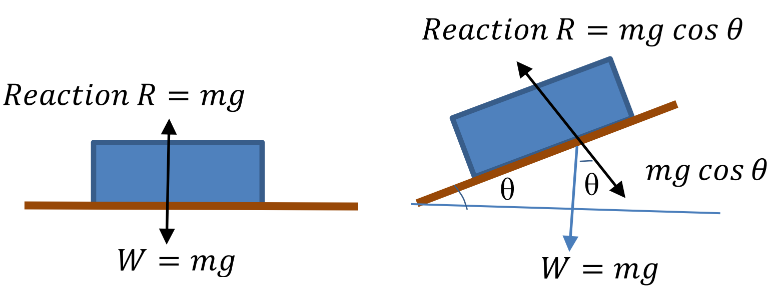
k is assigned a value of one (unity) when force is in Newtons (N), mass in kg and acceleration in m/s^2 hence:

$F = ma$ (viii)

Equation (viii) is the mathematical representation of Newton's second law of motion.

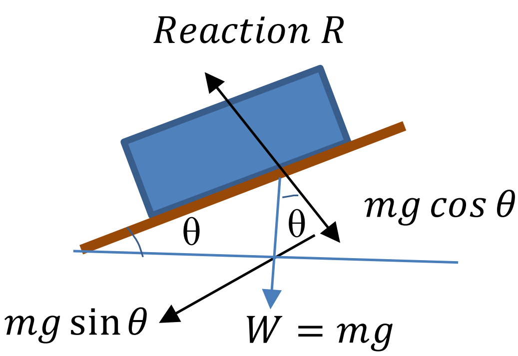
(iii) Newton's third law. This states that action and reaction are equal and opposite. A body exerts force on the surface it is resting on. Likewise, the surface exerts an equal but opposite force called reaction R on the body. If the surface is horizontal, the reaction is equal to the weight of the object. If on the other hand the surface is inclined, say at an angle ϑ to the horizontal, the reaction is equal to the component of weight that is perpendicular to the surface. For a mass m resting on a plane inclined at an angle ϑ therefore, the reaction R is given by;

$R = mg\cos\theta$ (ix)



Applications of action and reaction: rowing a boat - when water is pushed backwards, the boat moves forward; balloon flies - air gushes out from the rear and balloon moves in the opposite direction.

NOTE 1: The component of weight of the body along the plane inclined at an angle ϑ ($mg\sin\theta$) is responsible for motion down the plane.



If the body accelerates down the plane with an acceleration *a*, then by Newton's second law of motion (and assuming no frictional force exists), it follows that:

$mg\sin\theta = ma$ (x)

Considering that $\sin\theta = 0$ (minimum) when $\theta = 0$ and $\sin\theta = 1$ (maximum) when $\theta = 90^0$ it follows that the steeper the inclination, the higher the acceleration of the body.

NOTE 2: Equation (v) can be expresses as;

$F\Delta t = mv - mu$ (xi)

If Δt is very small (e.g., when a bat strikes a tennis ball) then;

$F\Delta t = impulse$ (xii)

$Impulse = mv - mu$ (xiii)

By equation (xiii) impulse of a force is equal to the change in momentum of an object when a force is applied for a very short period.

If the net force in equation (xi) is zero ($F = 0$) then;

$0 = mv - mu$ (xiv)

$mv = mu$ (xv)

It follows from equation (xv) that if no external force acts on a body, the final momentum is equal to the initial momentum (momentum is not changing with time) hence momentum is conserved

NOTE 3: Effective weight (force exerted on the floor) when travelling in a lift

- Lift stationary or moving with constant velocity, *v*: The force an object, say mass *m*, exerts on the floor of the lift is equal to its weight *W*, i.e.,

  $W = mg$ (xvi)
- Lift moving downwards with acceleration *a;* In this case, both the weight of the object (*mg*) and the external force on the object on account of an accelerating lift (*ma*) are in the same direction. The

effective force W' the object exerts on the floor of the lift therefore equals;

$W' = mg - ma = m(g - a).$ (xvii)

Equation (xvii) shows that an object weighs less than it really is if weighed while in a lift travelling downwards, and if $g > a$. If the acceleration of the lift equals the acceleration due to gravity i.e., $g = a$, then;

$W' = 0$ (xviii)

This means that the object does not exert any force on the floor of the lift and therefore appears weightless. If $g < a$, the speed of the lift is changing at a higher rate than that of the object hence W' is negative;

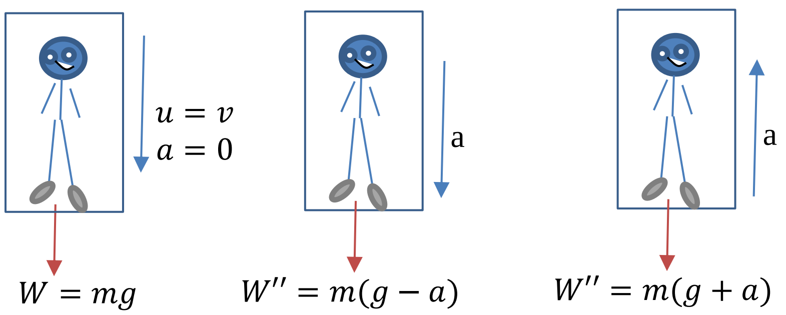
$W' < 0$ (xix)

The object in this case appears to move upwards relative to the lift.

- Suppose that the lift is moving upwards with constant acceleration a. The force on the object by virtual of gravity (mg) and that due to the external force (ma) are in opposite directions. The effective force W'' the object exerts downwards is therefore given by:

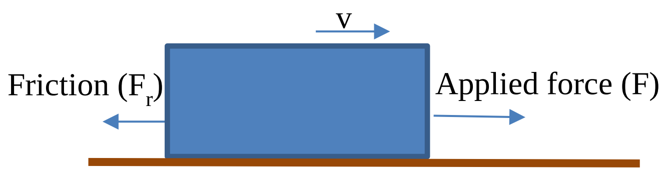
  $W'' = mg + ma = m(g + a).$ (xx)

  The object in this case appears to weigh more than it actually is.
- In a nutshell:



NOTE 4: Friction

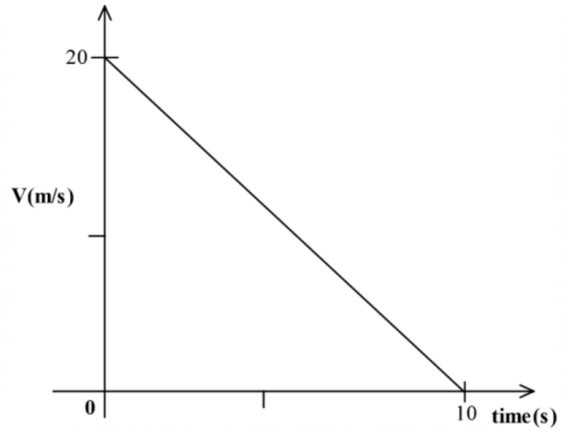
Friction is opposition to motion between two surfaces that are sliding or attempting to slide over each other. It acts in the direction opposite that of motion.



If frictional force equals the applied force, the body moves at a constant velocity (has zero acceleration). If applied force is greater than the frictional force, the body accelerates and if less than frictional force the body decelerates.
Advantages of friction– allows us to walk (cannot walk on slippery floor), write, drive (if road smooth, the vehicle skids).
Disadvantages of friction -causes wear and tear in moveable parts
Ways of reducing friction – oiling, using rollers and ball bearings.

To the question
$u = 20\, ms^{-1}$
$v = 0\, ms^{-1}$
$a = -\,2\, ms^{-2}$
$v = u + at$
$0 = 20 - 2t$
$20 = 2t$
$t = 10\, s$

(ii) Sketch the velocity-time graph for the motion of the bus up to the time it stopped. (2 marks)



(iii) Use the graph to determine the distance moved by the bus before stopping. (1 mark)

$$Distance = area\ under\ the\ graph = \frac{1}{2} \times 10 \times 20 = 100\, m$$

b) A car of mass 1000kg travelling at a constant velocity of 40ms^{-1} collides with a stationary metal block of mass 800kg. This impact takes 3 seconds before the two move together. Determine the impulsive force. (4 marks)

First determine the final velocity, say *v*

$momentum\ before\ collision\ = momentum\ after\ collision$

$(1000 \times 40) + (800 \times 0) = (1000 + 800)v$

$v = \frac{40000}{1800} = 22.22\ m/s$

then determine acceleration

$v = u + at$

$22.22 = 40 + 3a$

$a = -5.93\ m/s^2$

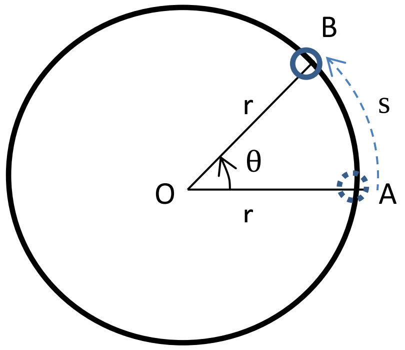
Then determine force:

$F = ma$

$F = 1800 \times -5.93 = -10674\ N$

11. State two factors that affect the angular velocity of a body moving in a circular path. (2 marks)

Circular motion refers to motion along a curved path e.g., a roundabout. Suppose an object moving along a circular path of radius *r* covers a distance *s* represented by the arclength AB (a section of the circumference of a circular path of a circle is referred to as arclength), sweeping an angle θ about the center *O.*



The angle θ swept by the particle (or the angle subtended at *O* by the arclength *s*) is called angular displacement. The SI unit of angular displacement is the radian, abbreviated as *rad*.

Now, circumference c of the circular path is given by;

$c = 2\pi r$ (i)

Equation (i) may be expressed as:

$\frac{c}{r} = 2\pi$ (ii)

If an object goes round the entire circumference, the curved distance (s) covered equals the circumference, i.e., $s = c$, while the angular displacement $\theta = 2\pi$. Bearing this in mind, equation (ii) may be generalised as;

$\frac{s}{r} = \theta$ (iii)

$s = r\theta$ (iv)

It is important to note that;

$2\pi\, rad = 360^0$ (v)

$1\, rad = \frac{360^0}{2\pi}$ (vi)

The rate of change of angular displacement with time is the angular velocity, ω, with units rad/s. Thus

$\omega = \frac{\theta}{t}$ (vii)

Angular velocity is a vector quantity with direction given by the right-hand rule: imagine gripping the axis of rotation with the right hand with fingers curled in the direction of rotation; the thumb points in the direction of angular velocity. Angular velocity is always perpendicular to the plane of motion.

The time taken by the object to complete 1 revolution is called period, *T*, with SI units seconds. Given that the angular displacement of an object after 1 revolution is equal to 2π , it follows that;

$\omega = \frac{2\pi}{T}$ (viii)

$T = \frac{2\pi}{\omega}$ (ix)

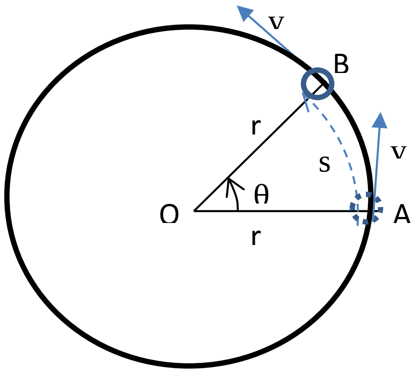
Frequency, *f*, with SI units Hertz, *Hz*, is the number of complete revolutions in unit time. It is also the reciprocal of period, i.e.

$f = \frac{1}{T}$ (x)

Using this in eq. (x) gives;

$\omega = 2\pi f$ (xi)

Linear velocity *v* (m/s) is the velocity the object would move at if for some reason it abruptly stopped moving in the circular path e.g., brakes failure of a car rounding a bend or moving in a roundabout, in which case the car would move in a straight line in accordance with Newton's first law of motion. The linear velocity is usually tangential to the circular path.



If for example an object moves along the arclength AB of length *s* with linear velocity *v,* from definition of velocity, it follows that:

$$v = \frac{arclength\ AB}{t} \qquad \text{(xi)}$$

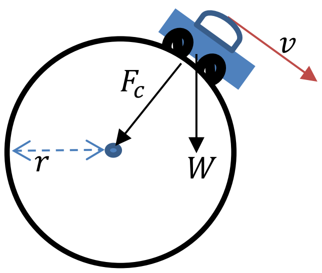
Since by equation (iv) $s = r\theta$, equation (xi) may be expressed as;

$$v = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) \qquad \text{(xii)}$$

From the definition of angular velocity, $\theta/_t$ equal angular velocity ω. Equation (xii) may therefore be written as:

$$v = r\omega \qquad \text{(xiii)}$$

For a body to sustain circular motion, an external force must act on it. This force is called centripetal force, F_c, and is directed towards the center of the circular path.



For a body, mass *m*, moving with velocity *v* along a circular path radius *r*, the magnitude of the centripetal force F_c is given by:

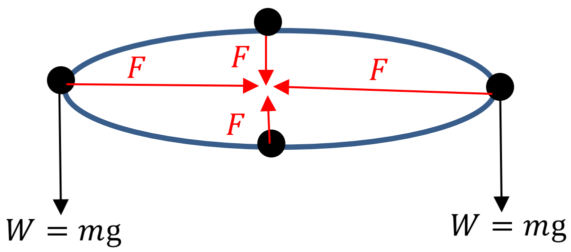
$$F_c = \frac{mv^2}{r} \qquad \text{(xiv)}$$

Where;

$$\frac{v^2}{r} = a\ (centripetal\ acceleration) \qquad \text{(xv)}$$

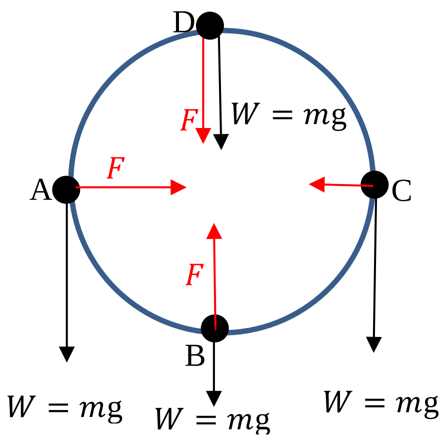
If the object is a car negotiating a roundabout, the centripetal force is provided by the frictional force between the tyres and the road, and if the object is a stone tied to a string and swirled in a circle, the centripetal force is provided by the tension in the string.

Suppose a stone, mass *m*, is tied to a string and swirled into a horizontal circle. Say the tension in the string is *F*. The weight of the stone, $W = mg$, acts vertically downward while the tension in the string is directed towards the center of the circular path

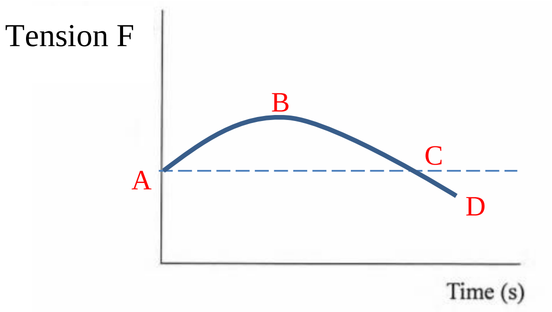


At all points of the horizontal circular path, the weight and the tension are at right angles. This means that the weight of the object has no effect on the tension on the string.

Suppose the stone is now swirled in a vertical circle.



At points A and C, the weight of the stone has no impact on the tension. At point B, the weight of the stone acts vertically downwards while the centripetal force acts vertically upwards. The tension in the string is therefore maximum at B. At point D, both the weight and centripetal force act in the same direction. The tension in the string is therefore minimum at this point.



NOTE: At points A and C (where the weight and tension are perpendicular to each other) the tension in the string is equal to the centripetal force, i.e.

$$F = F_c = \frac{mv^2}{r} \qquad \text{(xvi)}$$

To the question

$v = r\omega \Longrightarrow \omega = \frac{v}{r}$

Factors affecting angular velocity;

- linear velocity v
- radium of the circular path

12. Figure 4 shows two capillary tubes X and Y of different diameters dipped in mercury.

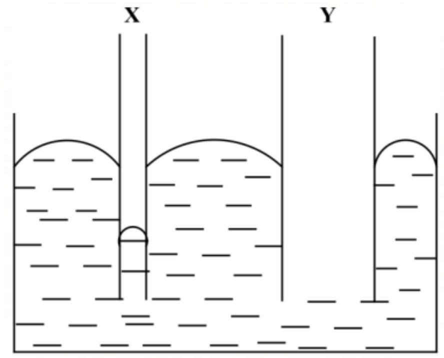
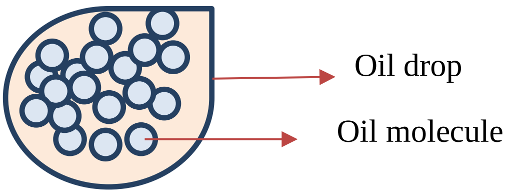


Figure 4

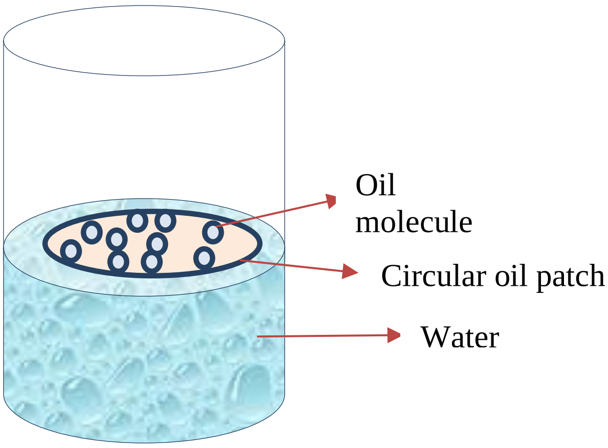
Complete the diagram to show the meniscus in Y

Surface tension, cohesive and adhesive forces

Imagine an oil drop is a balloon filled with a small cluster of balls, each ball representing an oil molecule.



When five such drops are placed in a waterbath, the oil spreads out in a thin layer on the surface of water.



The oil spreads on the surface of water because the adhesive force (force of attraction between non-identical molecules) between the water and oil molecules is greater than the cohesive force (force of attraction between identical molecules) between the oil molecules. The adhesive force reduces the water surface tension. The oil floats on water because it is less dense than water. Assuming that;

(i) Zero loss of oil molecules during the transfer process occurs hence volume (say *V*) of the oil remains constant

(ii) The oil forms a perfect circular patch (say radius r) and

(iii) the oil patch is one molecule thick (say *d*),

then, the volume of the five drops of oil is equal to the volume of the circular oil patch (essentially a cylinder that is one molecule thick), that is;

$$V = \pi r^2 d \qquad \text{(i)}$$

$$d = \frac{V}{\pi r^2} \qquad \text{(ii)}$$

The thichkness (diameter) of the molecule represents the size of the molecule Now, the area of the circular patch $A = \pi r^2$. Equation (i) may therefore be expressed as:

$$V = Ad$$

$$\Rightarrow A = \frac{V}{d} \qquad \text{(iii)}$$

NOTE 1: Surface tension is the tendancy of the surface of a liquid to behave like a stretched skin. Factors affecting surface tension of a liquid for example water include:

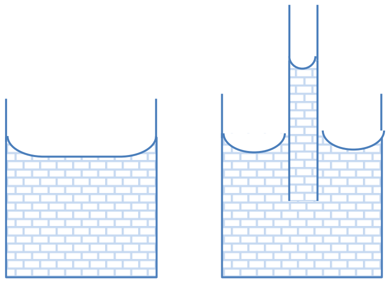
(i) Temperature; when the temperature rises, the speed of the molecules increases leading to a reduction in cohesive force hence a reduction in surface tension.

(ii) Impurities for example oil where the higher adhesive force between water an oil molecules reduce cohesive force between the water molecules.

(iii) Concentration; For solutions, the surface tension depends on the concentration of the solution. Surface tension increases if the solute is very soluble, for example salt. Salty water (hard water) has a higher surface tension compared to fresh water. If the solute is not very soluble e.g. soap or phenol, surface tension reduces.

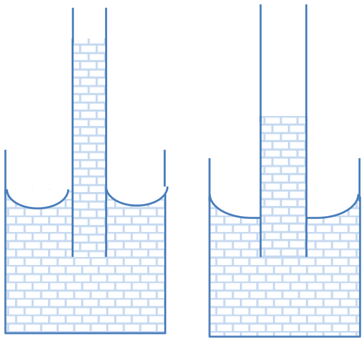
NOTE 2: Whether a liquid collects as a spherical drop or spreads on the surface where it is placed depends on the relationship between cohesive and adhesive forces. If the adhesive force is greater than the cohesive force, the liquid spreads e.g. oil on water, or water on glass (water is said to wet glass). If the cohesive force is greater, then the liquid collects as a spherical ball e.g. mercury on glass (mercury does not wet glas) or water on arrowroot leaf.

NOTE 3: Water rises up an open glass capillary tube dipped in water because the adhesive force between water and glass molecules is greater than the cohesive force between the water molecules (the tube is open so as to maintain equal pressure inside and outside the capillary tube).

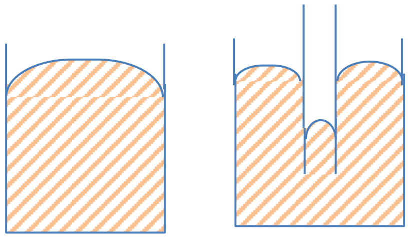


It is also for this reason that the meniscus curves upwards.

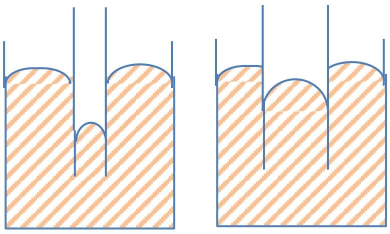
Hot water rise in a capillary tube is lower than cold water. This is because heat reduces the density of water and consequently its surface tension. The lower the surface tension, the lower the capillary rise. A narrower capillary tube on the other hand produces a higher rise of water level compared to a wider tube. This is due to increased adhesive force resulting from increased relative surface area inside the tube hence more water contact.



NOTE 4: The cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules. For this reason, mercury level in an open capillary tube dipped in merccury drops. It is also for this reason that the meniscus of mercury in glass curves downwards.



Increasing the temperature of mercury increases the kinetic energy of the molecules thereby reducing the surface tension. The depression of hot mercury down a capillary tube is therefore less than that of cold mercury. If capillary tubes of different diameters are used, the depression of mercury in the wider tube will be lower than in the narrower tube.



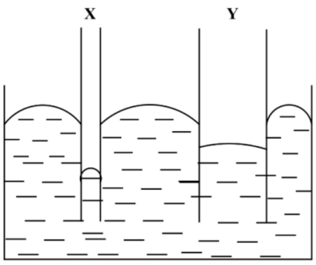
It is important to note that the behaviou of mercury in a capillary tube is opposite that of water. Colder water rises higher up the capillary tube while colder mercury has a more reduced rise (more depression). A narrower bore leads to a higher rise of water while a narrower bore leads to a lower rise (more depression) of mercury.

NOTE 5: Water with a lower cohesive force for example hot, fresh or soapy is better at cleaning as it wets clothes better.

NOTE 6: Mosquitoes lavae hang on the surface tension of stagnant water. One way of controlling the breeding of mosquitoes is reducing the surface tension of water for example by allowing it to flow freely. The resulting turbulence reduces the surface tension thus disrupting the breeding habitat.

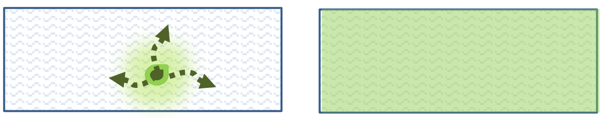
To the question now;

The level of mercury in the wider tube is higher than in the narrower tube.



13. In an experiment, a drop of black ink is introduced at the bottom of a container filled with water. It is observed that the water gradually turns black. State the effect on the observation when the experiment is carried out using water at a lower temperature. (1 mark)

Diffusion is the movement of particles from a region of high concentration to a region of low concentration on account of concentration gradient. If for instance a drop of green ink is added to water, the ink gradually spreads out. The water eventually attains a uniform green colour as the diffusing particles take up spaces between its molecules.



Diffusion is more pronounced in liquids and gases (fluids) for the reason that their molecules are energetic enough to undergo translational random motion (Brownian motion). There are various factors that affect the rate of diffusion in fluids:

(i) Concentration gradient; the higher the concentration difference between the solvent (fluid at low concentration) and the solute (fluid at higher concentration), the higher the rate of diffusion. A more concentrated green ink would change the colour of the entire water faster than a less concentrated one.

(ii) Temperature; When the temperature of the solvent or the solute increases, the rate of diffusion also increases. An increase in temperature increases the random movement of both the solute and solvents hence the increased rate of diffusion.

(iii) Mass of the solute particles; Heavy particles move more slowly leading to low rate of diffusion.

(iv) Density of the solvent; A denser solvent slows down the rate of diffusion.

Examples of diffusion in daily life;

- Smell of perfume
- When a soda is opened, CO_2 diffuses out
- Using teabags – tea diffuses in hot water
- Sugar can dissolve in tea without stirring
- Milk mixes with water without stirring.

To the question

If temperature is reduced, the rate of diffusion reduces hence the water turns black more slowly.

14. Figure 5 shows two identical springs arranged side by side and supporting a weight of 50 N.

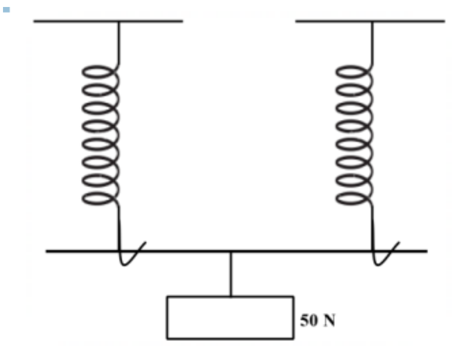


Figure 5

When the same weight is supported by one of the springs above, it produces an extension of 1 cm. Determine the effective spring constant of the arrangement in Figure 5. (3 marks)

Some materials increase in size when stretched, decrease in size when compressed, and snap back to their original shape when the deforming force is withdrawn. Such materials are said to be elastic. If the force exceeds some threshold value however, the materials do not regain their original shape after the deforming force is withdrawn. The point at which this occurs is referred to as the elastic limit (elastic limit is defined as the point beyond which an elastic material does not regain its shape after the deforming force is withdrawn). Examples of elastic materials are rubber, steel, and coils (springs). Other materials are permanently deformed as soon as a deforming force is applied. If such a material is expanded for instance, it retains the expanded size even after the expanding force is withdrawn. Such materials are called plastics. A good example of plastic is a polythene bag (*juala*). It is important to note that all elastic materials turn into plastic when deformed beyond their elastic limits.

NOTE 1: Applications of elastic materials

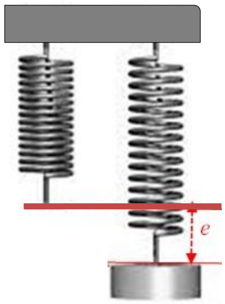
- Trampoline – jump on it, it stretches, then regains its shape.
- Elastic band – used for fastening for example bundles of paper.
- Springs – used in spring balances used for weighing.
- Clothes – that pair of skinny jeans that fits like a second skin is made using an elastic material

NOTE 2: Applications of plastics

- Used for packaging
- Casing of appliances such as phones, toys, cars etc.

NOTE 3: Hooke's law

Hooke's law states that extension of an elastic object is directly proportional to the force applied to it so long as the elastic limit is not exceeded. Say for example a force F is applied to a spring such that it extends by an amount e (equal to final length, say L, less the original length, say L_0, i.e., $L - L_0$).



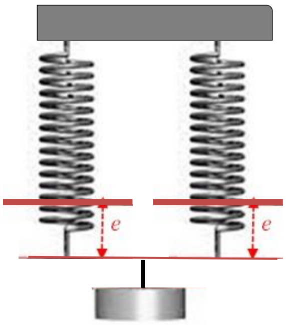
$F = W = mg$

It follows that by Hooke's law,

$F = k(L - L_0) = ke$ (i)

$\frac{F}{e} = k$ (ii)

k is a constant of proportionality called spring constant; it is a measure of the stiffness of a spring (coil). A spring that stretches easily has a smaller spring constant compared to a stiffer one and as such reaches the elastic limit sooner. Consider two identical springs, length L and spring constants k_1 and k_2 respectively. Suppose that when the two spring are arranged in parallel and a force F applied to both springs, each spring stretches by an amount e.



$F = W = mg$

The force F is divided equally between the springs and as such each spring supports a force $F/2$. It therefore follows that by Hooke's law;

Spring 1: $\frac{F}{2} = k_1 e$ (iii)

Spring 2: $\frac{F}{2} = k_2 e$ (iv)

Total force: $F = \frac{F}{2} + \frac{F}{2} = k_1 e + k_1 e$ (v)

$F = (k_1 + k_2)e$ (vi)

The value $k_1 + k_2$ represents the effective spring constant of the arrangement.

Considering that the two springs are identical, then;

$k_1 = k_2 = k$ (vii)

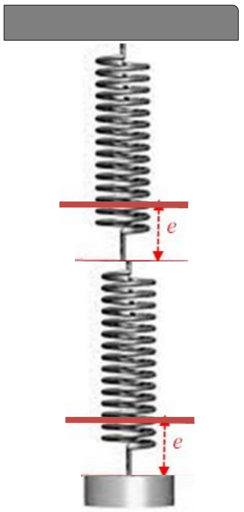
$\Longrightarrow k_1 + k_2 = 2k$ (viii)

Equation (viii) shows that if two springs in parallel are connected in such a way that they share a common load, the effective spring constant doubles.

Suppose that the two springs are now connected in series and after the load is attached, each spring stretches by an amount e_1 and e_2 respectively. Since both loads are supporting the same load *F*, then by Hooke's law, the springs extends by:

Spring 1: $e_1 = {}^{F}/_{k_1}$

Spring 2: $e_2 = {}^{F}/_{k_2}$



$F = W = mg$

If e be the total extension, then;

$e = e_1 + e_2$

$e = \frac{F}{k_1} + \frac{F}{k_1} = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$ (ix)

Given $k_1 = k_1 = k$, then;

$\frac{e}{F} = \frac{2}{k}$ (x)

$\Longrightarrow F = \left(\frac{1}{2}k\right)e$ (xi)

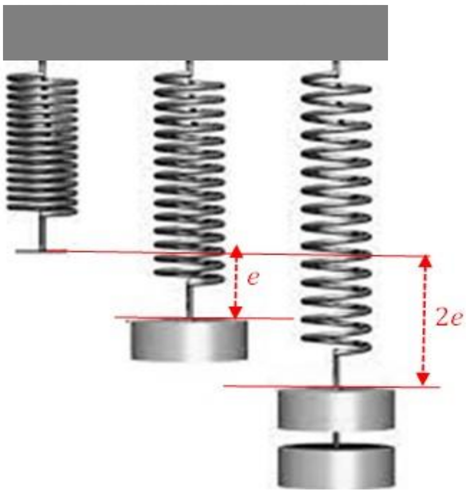
$\frac{1}{2}k$ is the effective spring constant. For two identical springs connected in series therefore, the spring constant reduces by half.

There are various factors that affect the spring constant:

(i) Length: Spring constant is inversely proportional to length. An increase in length leads to a decrease in the spring constant and vice versa.

(ii) Thickness of the wire making the spring. A thicker wire makes a stiffer spring with a larger spring constant.
(iii) Cross sectional area (diameter) of the spring: A wider spring is less stiff compared to a narrower spring. The greater the cross-sectional area of a spring therefore, the smaller the spring constant.
(iv) Number of active coils (turns) that are free to expand. A spring with relatively more turns is springier hence has a smaller spring constant.
NOTE 1: Experiment to prove Hooke's law and determine the spring constant;

- Suspend a spring and measure its length (original length)
- Suspend first mass, measure the new length
- Determine the extension $(new\ length - original\ length)$
- Keep adding more masses and noting the new extension (e) every time.



- Plot a graph of weight $(F = mg)$ against extension (e) – it should be a straight line where the spring obeys Hooke's - and determine the gradient (k) law.
- The gradient of the graph represents the string constant.

NOTE 2: Hooke's law can also be defined in terms of stress and strain as: the stress applied to a material within the elastic limit is proportional to the strain, i.e.

$stress \propto strain$ (iii)

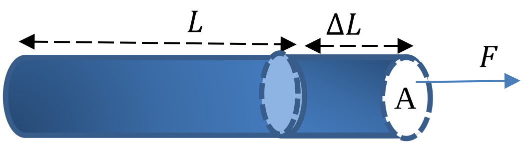
Stress refers to the force acting on a unit cross-sectional area of an object.

$stress = \frac{Force}{Area}$ (iv)

If the stress causes the length of the object to change, the strain is defined as the ratio of the change in length to the original length;

$Strain\ = \frac{chnge\ in\ length}{original\ lenth}$ (v)

Now, consider a wire of length L and cross-sectional area A. Suppose a force F acts on the wire such that its length changes by ΔL.



It follows that;

$Stress = \frac{F}{A}$ (vi)

$Strain = \frac{\Delta L}{L}$ (vii)

From Hooke's law;

$\frac{F}{A} \infty \frac{\Delta L}{L}$ (viii)

$\frac{F}{A} = \gamma \frac{\Delta L}{L}$ (ix)

Where γ is the constant of proportionality called modulus of elasticity or young's modulus. Because $\Delta L = e\ (extension)$, equation (ix) may be written as;

$\frac{F}{A} = \gamma \frac{e}{L}$ (x)

$\gamma = \frac{L}{A} \frac{F}{e}$ (xi)

It is important to note that if the ratio of length of the wire to its cross-sectional area is unity, i.e., $\frac{L}{A} = 1$, Young's modulus (modulus of elasticity) and spring constant are equal.

To the question

for a parallel arrangement the spring constant is double the spring

$k = k_1 + k_1 = 2k_1$

$F = k_1 e$

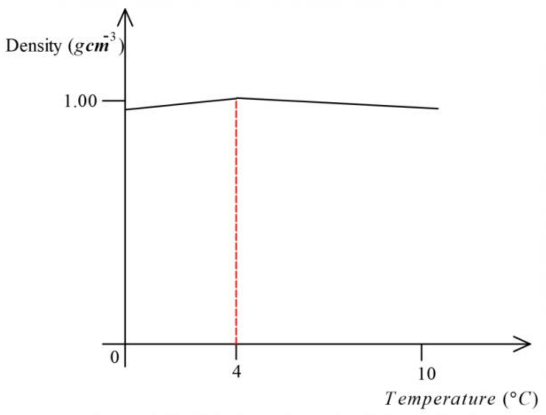
$50 = k_1 \times 0.01$

$k_1 = 5000\ N/m$

$k = 2k_1 = 2 \times 5000 = 10000 N/m$

15. On the axes provided, sketch a graph of density against temperature for water between 0°C and 10°C

Water behaves abnormally when heated from 0 ^{0}C to 4 ^{0}C in that it contracts as opposed to expand. Above this threshold temperature, the water starts expanding as expected. From the definition of density, a reduction in volume, mass constant, leads to an increase in density. The density of water therefore increases between 0 and 4 ^{0}C after which the density starts to reduce.



16. Figure 6 shows a graph of temperature against time for a pure molten substance undergoing cooling.

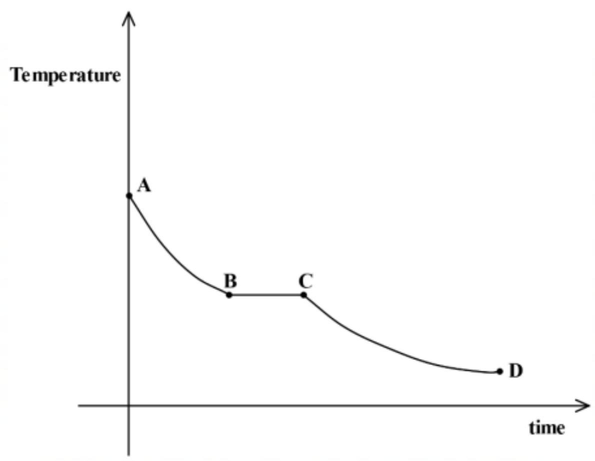


Figure 6

Explain what happens to the substance in region BC (2 marks)

In physics, calorimetry involves the measurement of the amount of heat transferred between objects. Heat (net heat) flows from hot body to cold body it is in thermal contact with until the two bodies acquire the same temperature. The bodies are then said to be in thermal equilibrium. If the two systems are isolated (no heat lost to the surrounding), then at thermal equilibrium, the heat lost by the hot body equals the heat gained by the cold body. There are various terms associated with calorimetry:

- Heat/thermal capacity (C): This refers to the amount of heat required to produce a unit rise in temperature in a substance. This means that if an amount of heat equal to Q is added to a substance such that its temperature rises by $\Delta\vartheta$, then;

  $C = \frac{Q}{\Delta\theta}$ (i)

$Q = C\Delta\theta$ (ii)

- Specific heat capacity (c)**:** This refers to the heat required to change the temperature of a unit mass (1 kg) of a substance by 1 unit (1^0C or 1 K). If for example the temperature of an object of mass m changes by $\Delta\theta$ when heat equal to Q is added, the specific heat capacity c is given by:

  $c = \frac{Q}{m\Delta\theta}$ (iii)

  Equation (iii) is often expressed as;

  $Q = mc\Delta\theta$ (iv)

  Water has a high specific heat capacity and as a result can absorb a relatively large amount of heat before reaching the boiling point. It is for this reason that water is used as a coolant.

- Latent heat; This refers to the amount of heat required to change the state of matter. Latent heat is also referred to as hidden heat because it does not lead to an increase in temperature. The amount of heat required to change the state of a unit mass of a solid to a liquid (or liquid to solid) without a change in temperature is referred to as the latent heat of fusion. For example, if ice at -10^0C is heated, its temperature increases until it attains a temperature of 0^0C. The temperature then remains constant even with the addition of more heat. During this time however, the ice melts and turns to water at 0^0C. If heating stopped when the temperature of ice was at 0^0C, the ice would not have melted. This means that heat is required to melt the ice, and that heat is the latent heat of fusion. The heat is used to weaken the cohesive force between the ice molecules as opposed to raising the temperature. The temperature at which substances melt (melting point) is different for different substances. It is important to note that;
  - Presence of impurities lower the melting point.
  - Increase in pressure lowers the melting point
  - The higher the cohesive force the higher the melting point. For example, ice aside, the melting point of wax is lower than that of metal.

  Continued application of heat to water at 0^0C causes its temperature to rise up to 100^0C at normal atmospheric pressure. The increase in temperature remains constant as the water changes to vapour. The heat required to change the state of a unit mass of a liquid to gas (or gas to liquid) without a change in temperature is called latent heat of vaporization. Note that various factors affect the boiling point. These include;
  - Pressure on the liquid. Water for instance has a higher boiling point at sea level where the atmospheric pressure is high compared to the top of a mountain where the pressure is low.

- Impurities in the liquid raise the boiling point
- The lower the cohesive force, the lower the boiling point.

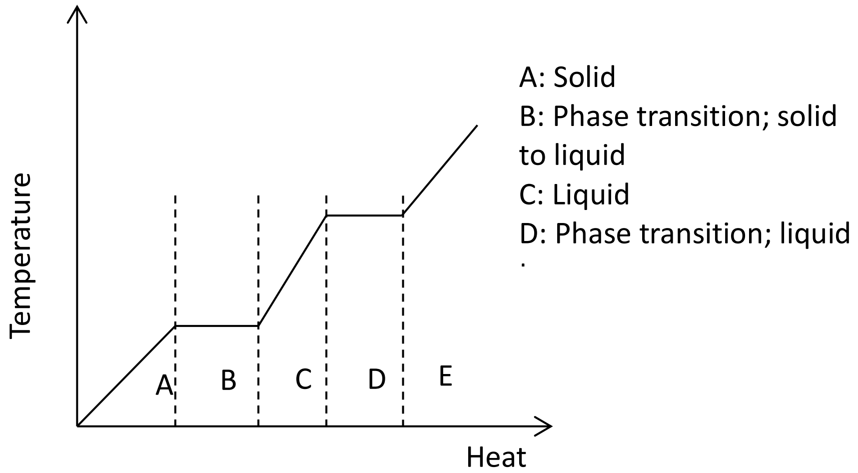
Thus, if Q be the heat required to change the state of a substance of mass m without there being a change in temperature, the latent heat L is given by;

$$L = \frac{Q}{m} \qquad \text{(v)}$$

$$Q = mL \qquad \text{(vi)}$$

Vaporization is associated with a cooling effect. If for example methylated spirit is poured on the palm of the hand, the palm feels cold. Methylated spirit has a low latent heat of vaporization and therefore vaporizes easily. Heat drawn from the palm is used to vaporize the methylated spirit hence the palm feels cold. In nature, sweating causes cooling effect when the sweat evaporates as it draws heat of vaporization from the body. This means that if the sweat does not evaporate (for example in very humid and non-windy areas), the cooling effect is minimal. It is for this reason that sportswear is made of materials that allow sweat to pass through and consequently evaporate (as opposed to an absorbent material which keeps the skin dry instead of allowing the sweat to evaporate).

NOTE 1: Phase transition from solid to vapour can be represented by a temperature-heat graph.



NOTE 2: Experiment to determine specific heat capacity of a solid by the method of mixtures.

- The solid is weighed and its mass m_s noted
- To heat the solid, the solid is suspended with a thread and submerged in a beaker containing boiling water.
- Meanwhile, an empty dry calorimeter is weighed and the mass, say m_{ce} noted.

- The calorimeter is then half-filled with cold water, weighed and mass m_{cw} and temperature T_{wi} noted.
- Mass of water in the calorimeter is then obtained as: $m_w = m_{cw} - m_{ce}$
- When the solid and the boiling water attain a steady temperature, the steady temperature, say T_{si} is measured.
- The solid is then removed from the boiling water, gently dipped in the calorimeter, and the contents well stirred. The final highest temperature reached, say T_f, is noted
- Assuming that no heat is lost to the surrounding, the heat lost by the solid should be equal to the heat gained by the water and calorimeter, i.e.,

  $$m_s c_s(T_{si} - T_f) = m_w c_w(T_f - T_{wi}) + m_{ce} c_c(T_f - T_{wi})$$

  Where c_s, c_w *and* c_c represent the specific heat capacities of the solid, water and calorimeter respectively. Thus,

  $$c_s = \frac{[m_w c_w + m_{ce} c_c](T_f - T_{wi})}{m_s(T_s - T_f)}$$

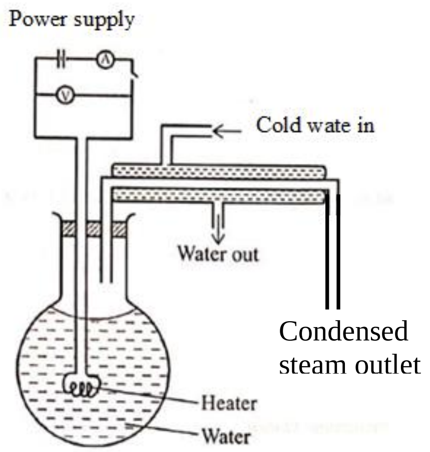
NOTE 3: Experiment to determine latent heat of vaporization

Method 1

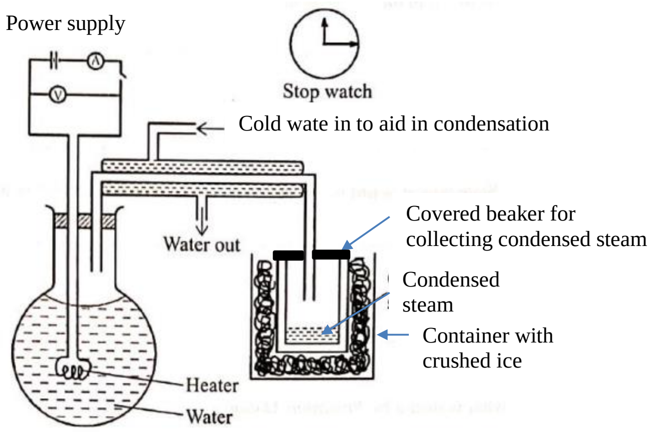
An electric heater converts electrical energy to heat. If the heater is connected across a voltage *V* such that a current, *I,* flows through the heater for a time *t*, the heat energy *E* produced is equal to;

$E = VIt$

Experimental set-up for the experiment is as shown.



- The power supply is switched on and the voltage *V* and current *I* recorded.
- As the water heats up, an empty beaker for collecting condensed steam is weighed and the mass m_1 recorded.
- The beaker is then placed in a container lined with crashed ice.
- As soon as the water being heated starts to boil, the clock is started and the beaker in crashed ice placed under the condensed steam outlet.



- Once an appreciable amount of condensed steam (steam that has turned into water) is collected, the time is noted, say *t*, and the beaker together with the condensed steam weighed and the mass, say m_2 recorded.
- The mass *m* of the condensed steam is obtained as;
- $m = m_2 - m_1$
- Assuming no heat is lost, the heat supplied by the heater (Vit) from when the water started boiling was used to generate the steam of mass *m*. If *L* be the specific latent heat of vaporization, then;

  $$Vlt = mL$$

  $$L = \frac{VIt}{m}$$

Method 2:

A beaker is filled with water, placed on a weighing balance and heated with an immersion heater.

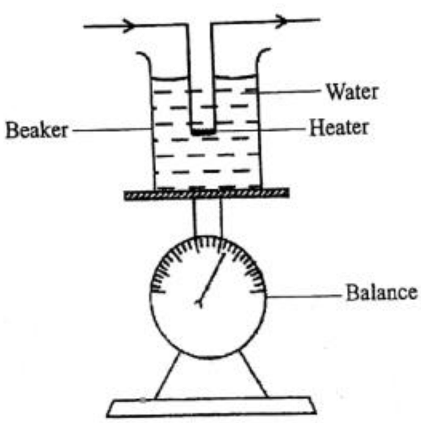


Figure 14

The voltage (*v*) across, and current (I) through, the heater is noted. Immediately the water starts boiling, the reading on the balance m_1 is noted and the stop watch started. The final reading on the balance m_2 is then noted after a time *t* seconds.

As soon as the water starts boiling, temperature becomes constant and the heat supplied by the heater (electrical energy, E) is used to turn water into steam. If no heat is lost, then;

$E = mL$

Now,

$E = VIt$

m is the mass of the steam generated given by

$m = m_2 - m_1$

Hence

$VIt = (m_2 - m_1)L$

$L = \frac{VIt}{(m_2 - m_1)}$ (*)

Note: The wattage or power (P) of the heater is given by;

$P = VI$

Equation (*) may therefore be expressed as;

$$L = \frac{Pt}{(m_2 - m_1)}$$

To the question now;

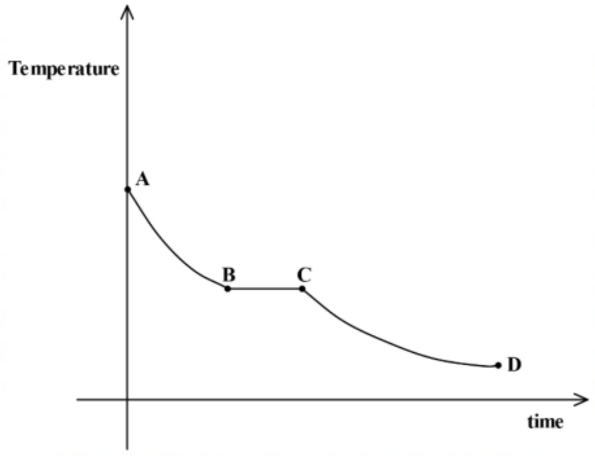


Figure 6

Region BC: The substance undergoes change of state from molten state to solid as no change in temperature occurs.

17 a) Define the term "specific latent heat of fusion" (1 mark)

Quantity of heat required to change a unit mass of the material from solid state to liquid without change in temperature.

b) Ice of mass 5g at a temperature of −10℃ is immersed into 10.5g of hot water at 100℃ in a container of negligible heat capacity. All the ice melts and the final temperature of the mixture is 40℃. Assuming there are no heat losses to the surrounding and taking specific latent heat of fusion for ice as L_f. (C_{water} = 4200 Jkg^{-1} K^{-1} and C_{ice} = 2100 Jkg^{-1} K^{-1}). Determine the:

(i) heat lost by the water. (3 marks)

$$Q = m_{water} C_{water} \Delta\theta$$
$$Q = 0.0105 \times 4200(100 - 40)$$
$$Q = 2646\, J$$

(ii) heat gained by ice from −10℃ to 0℃

$$Q = m_{ice} C_{ice} \Delta\theta$$
$$Q = 0.005 \times 2100 \times 10$$
$$Q = 105\, J$$

iii) heat required to melt the ice in terms of Lf (1 mark)

$$Q = m_{ice} L_f = 0.005 L_f$$

(iv) heat gained by the melted ice. (2 marks)

$$Q = m_{water} C_{water} \Delta\theta$$
$$Q = 0.005 \times 4200 \times 40$$
$$Q = 840\, J$$

(v) specific latent heat of fusion. (3 marks)

$$heat\ lost\ by\ hot\ water = heat\ gained\ by\ ice(-10℃\ to\ 0℃) + melting\ ice + heatgained\ by\ mealted\ ice\ (0 - 40℃\ )$$
$$2646 = 105 + 0.005Lf + 840$$
$$L_f = 340,200\, J$$

18. Figure 8 shows two pieces of ice A and B trapped using wire gauze in a larger beaker containing water.

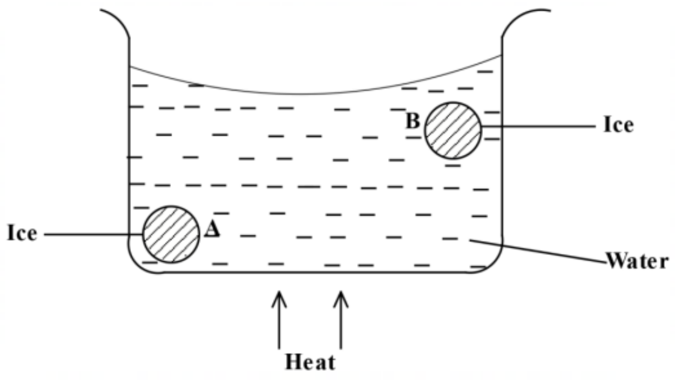


Figure 8

Heat is supplied at the center of the base of the beaker as shown. State the reason why B melted earlier than A. (1 mark)

Heated water at the bottom becomes less dense which rises to the top. Hence ice B melts earlier than A.

19. In a Physics experiment, a student filled a burette with water up to a level of 15ml. The student ran out 3 drops of water each of volume 2cm^3 from the burette into a beaker. Determine the final reading of the burette. (3 marks

Initial burette reading = 15ml

1 ml = 1 cm^3 hence 15 ml = 15 cm^3

Volume of drops = 3 × 2 =6 cm^3

New burette reading = 15- 6 = 9 cm^3

END